

# Performance Analysis, Scheduling and Synthesis of Embedded Systems

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DENMARK



DaNES



CENTER FOR INDLEJREDE SOFTWARE SYSTEMER

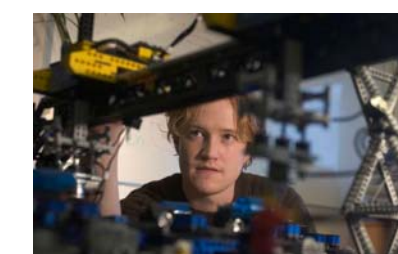
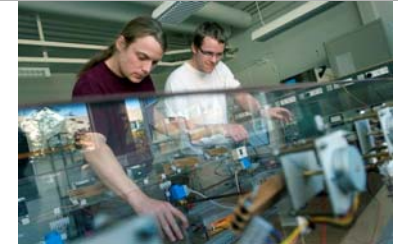
# CISS in Numbers



- National ICT Competence Center 2002:

**31,5** MDKK Ministry  
**8,5** MDKK North Jutland  
**7,5** MDKK Aalborg City  
**16,00** MDKK Companies  
**16,00** MDKK AAU

- **45** projects
- **20** CISS employees
- **25** CISS associated researcher at 3 different research groups at AAU!
- **>20** industrial PhDs



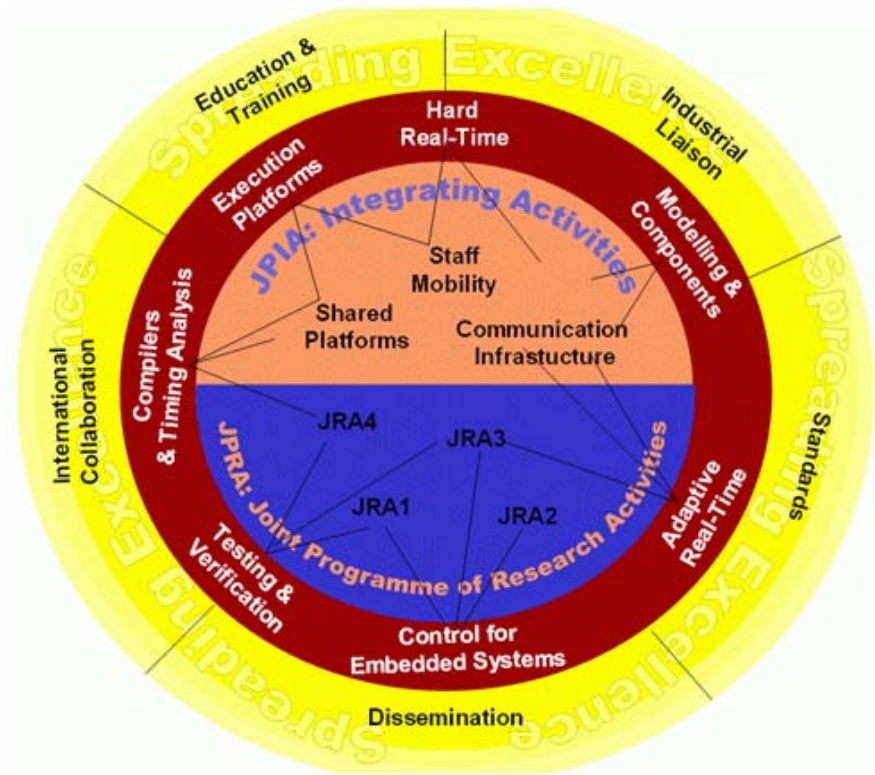
# European Projects



- **ARTIST2** (NoE FP6)
  - coordinator for Testing & Verification Cluster



- **ARTIST Design** (NoE FP7)
  - kick-off meeting end of January
  - co-coordinator of Modeling and Validation w Tom Henzinger)
- Other new STREPs (FP7)  
**Quasimodo, Multiform**

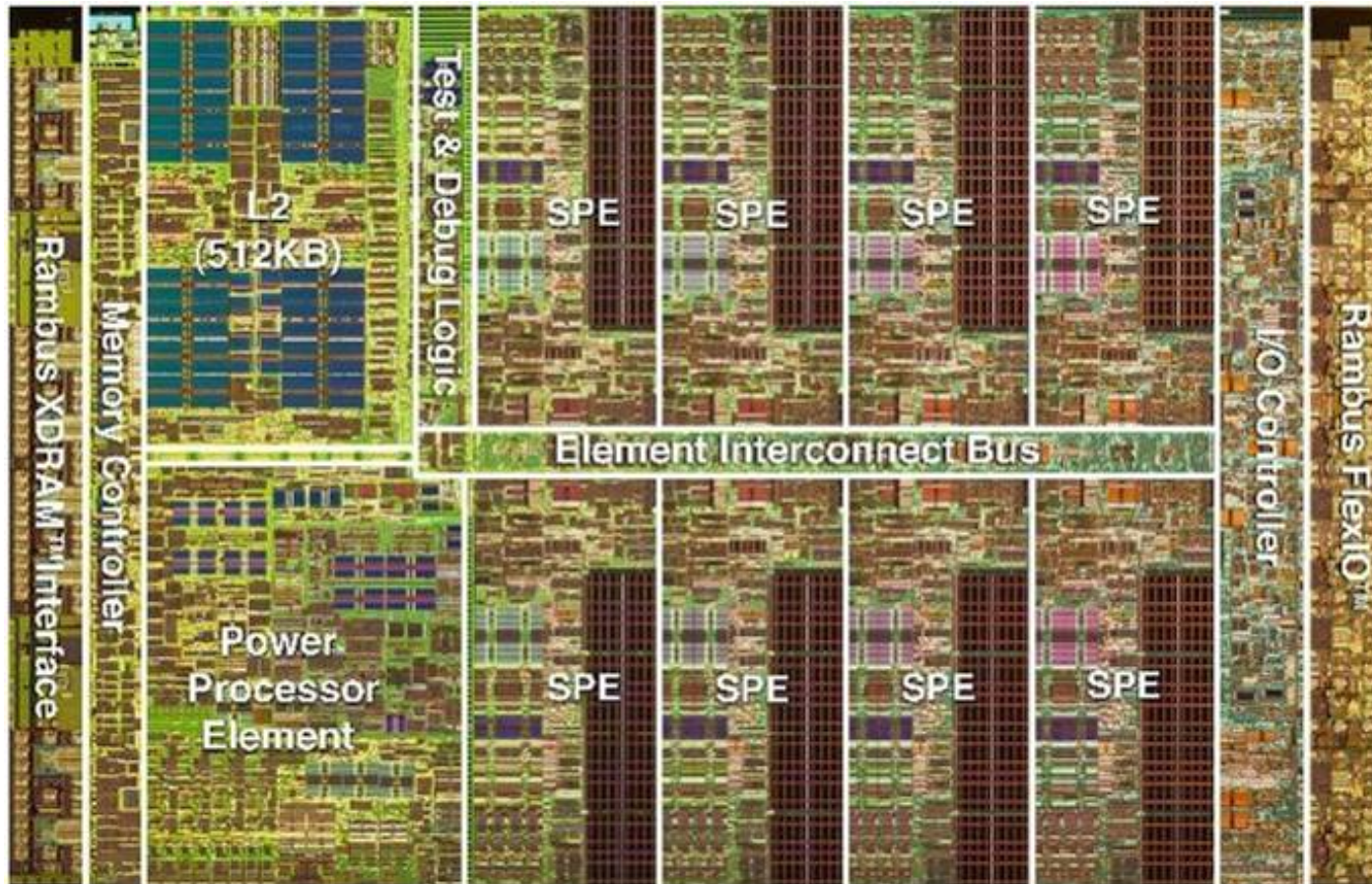




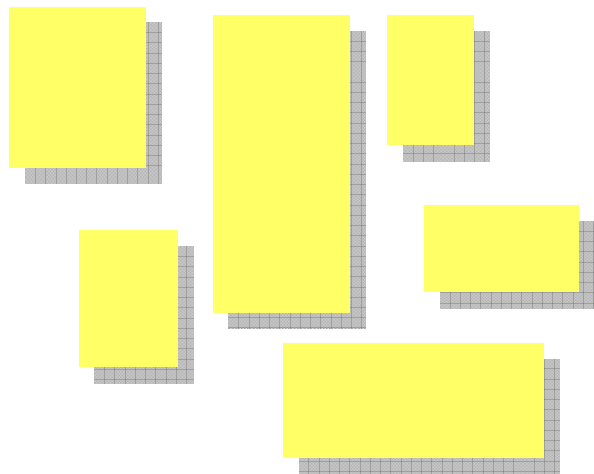
# Motivation – MPSoC



## CELL processor

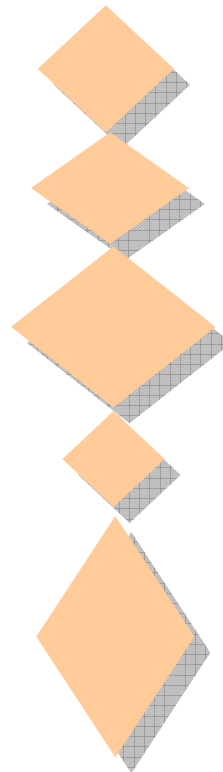


# Scheduling... in ES

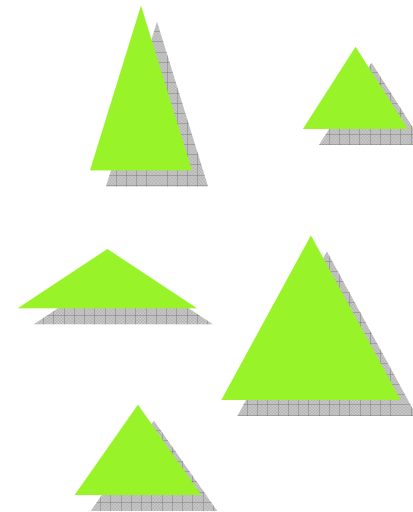


## Tasks:

Computation times  
Deadlines  
Dependencies  
Arrival patterns  
**uncertainties**



Scheduling Principles (OS)  
EDF, FPS, RMS, DVS, ..

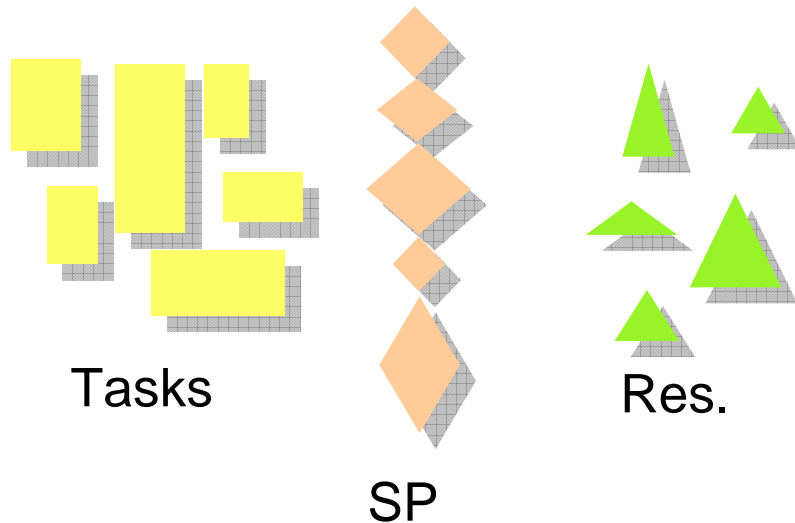


## Resources

Execution platform  
PE, Memory  
Networks  
Drivers  
**uncertainties**



# Issues



- **Schedulability Analysis**
  - Verify that given SP ensures deadlines.

- **Performance Evaluation**
  - Estimate resources (e.g. energy) required by given SP.
- **Scheduling & Synthesis**
  - Synthesize (optimal) SP ensuring given objective.
  - *Scheduling*: SP controls everything (including ex.time).
  - *Synthesis*: scheduling under uncertainties (e.g. execution time, availability of resources).

# Approach – TA



Task

UPPSALA UNIVERSITET AALBORG UNIVERSITY DENMARK

UPPAAL 4.0

Start Appr Safe Stop

```
while (i < len) {
  list[i] = list[i + 1];
  i++;
}
```

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More information at <http://www.uppaal.com>

UPPAAL 4.0.2 (rev. 2491), August 2006.

- Schedulability
  - Verify that given SP ensures deadlines.

- Performance Evaluation
  - Estimate resources (e.g. energy) required by given
- Scheduling Synthesis
  - (optimal) SP given objective.
  - Scheduling: SP controls

CLASSIC

CORA

TIGA

**TALK:**

What can we do?

What can we do **efficiently**?

What **can not** be done?

What would we **like to** do?

# The UPPAAL Team



## @UPPsala



- Wang Yi
- Paul Pettersson
- John Håkansson
- Anders Hessel
- Pavel Krcal
- Leonid Mokrushin
- Shi Xiaochun

## @AALborg



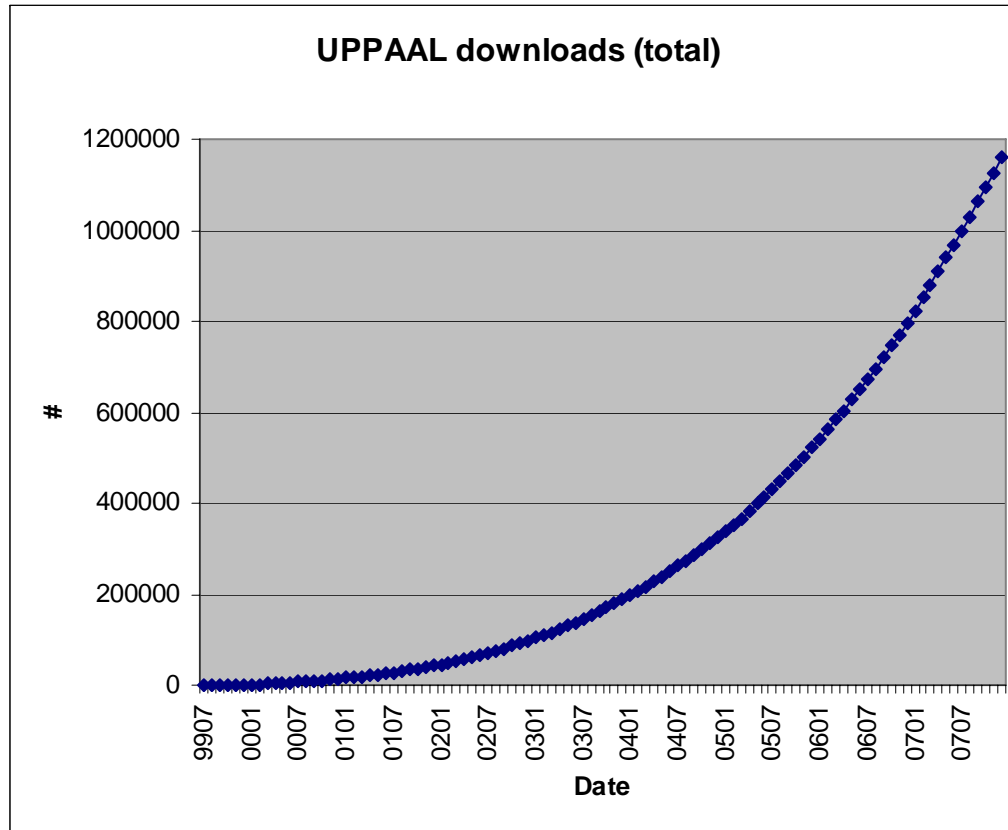
- Kim G Larsen
- Gerd Behrman
- Arne Skou
- Brian Nielsen
- Alexandre David
- Jacob I. Rasmussen
- Marius Mikucionis
- Thomas Chatain

## @Elsewhere

- Emmanuel Fleury, Didier Lime, Johan Bengtsson, Fredrik Larsson, Kåre J Kristoffersen, Tobias Amnell, Thomas Hune, Oliver Möller, Elena Fersman, Carsten Weise, David Griffioen, Ansgar Fehnker, Frits Vandraager, Theo Ruys, Pedro D'Argenio, J-P Katoen, Jan Tretmans, Judi Romijn, Ed Brinksma, Martijn Hendriks, Klaus Havelund, Franck Cassez, Magnus Lindahl, Francois Laroussinie, Patricia Bouyer, Augusto Burgueno, H. Bowmann, D. Latella, M. Massink, G. Faconti, Kristina Lundqvist, Lars Asplund, Justin Pearson...



# “Impact”



## Google:

UPPAAL: 134.000

SPIN Verifier: 242.000

nuSMV: 57.700

> 2.900

Google Scholar Citations

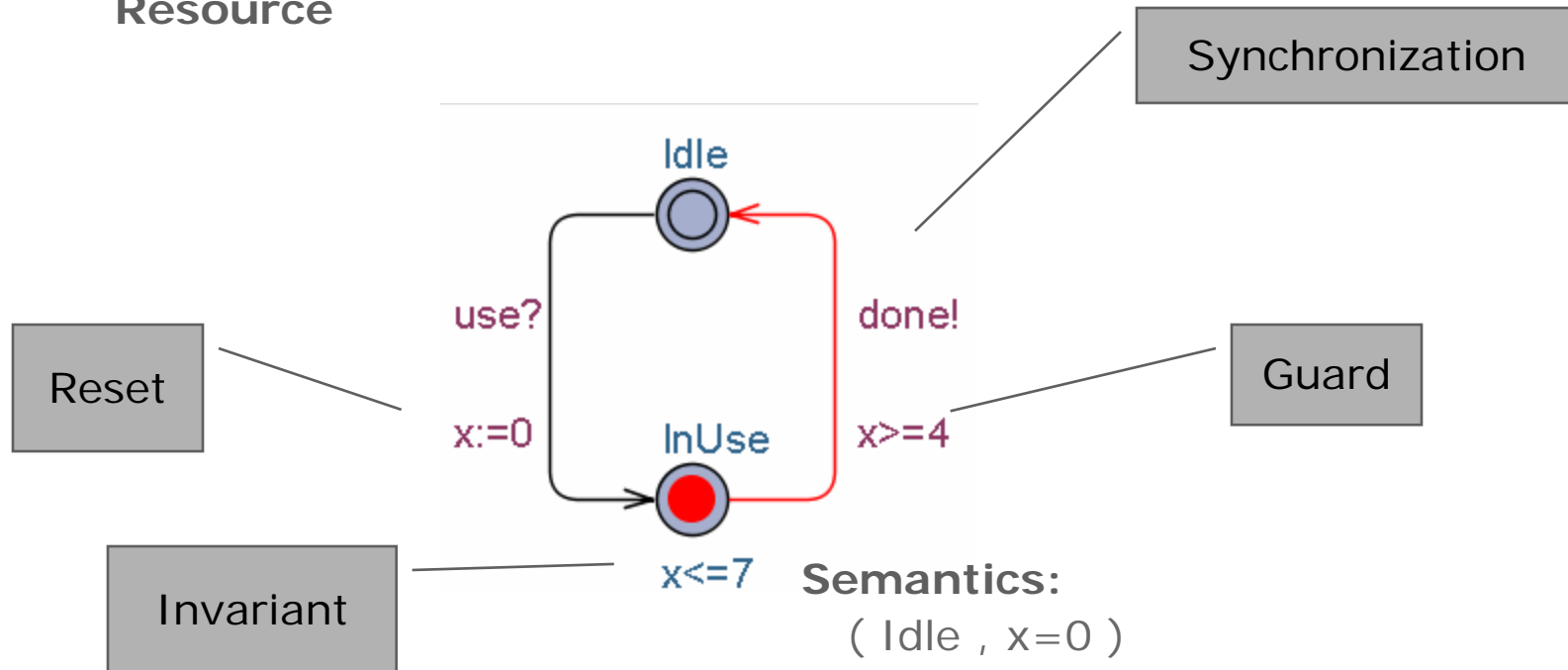
(Rhapsody/Esterel < 5.000)

# Timed Automata

[Alur & Dill'89]



Resource



Semantics:

( Idle , x=0 )

→ ( Idle , x=2.5 )

d(2.5)

→ ( InUse , x=0 )

use?

→ ( InUse , x=5 )

d(5)

→ ( Idle , x=5 )

done!

→ ( Idle , x=8 )

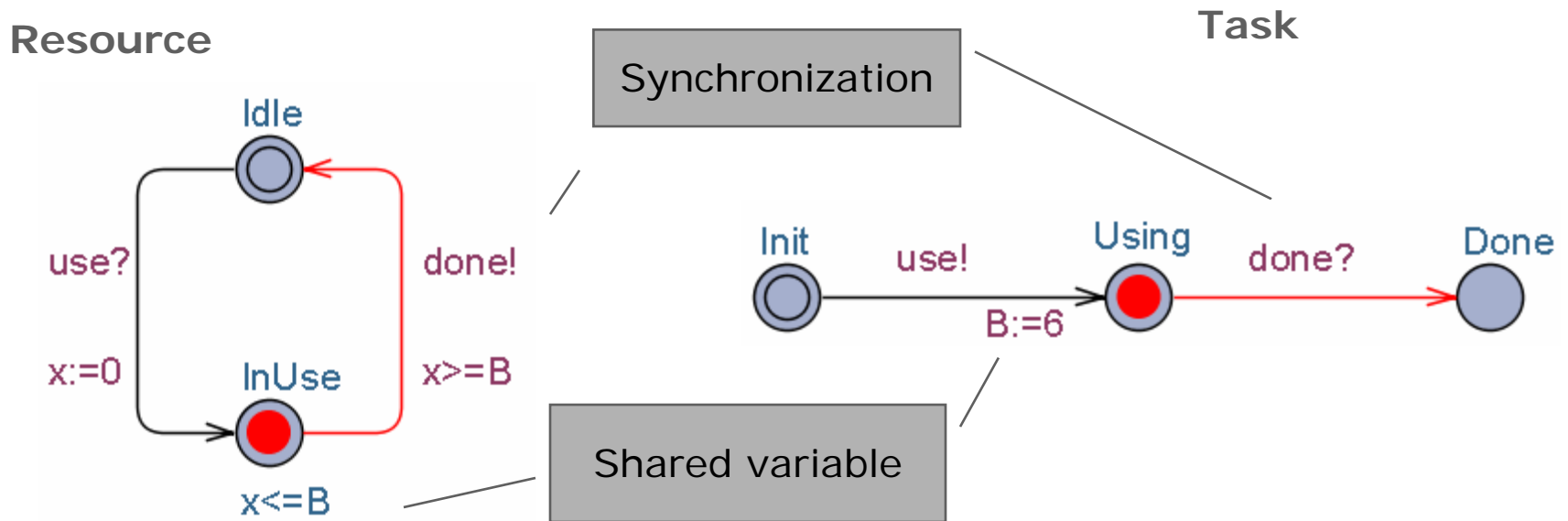
d(3)

→ ( InUse , x=0 )

use?



# Composition



## Semantics:

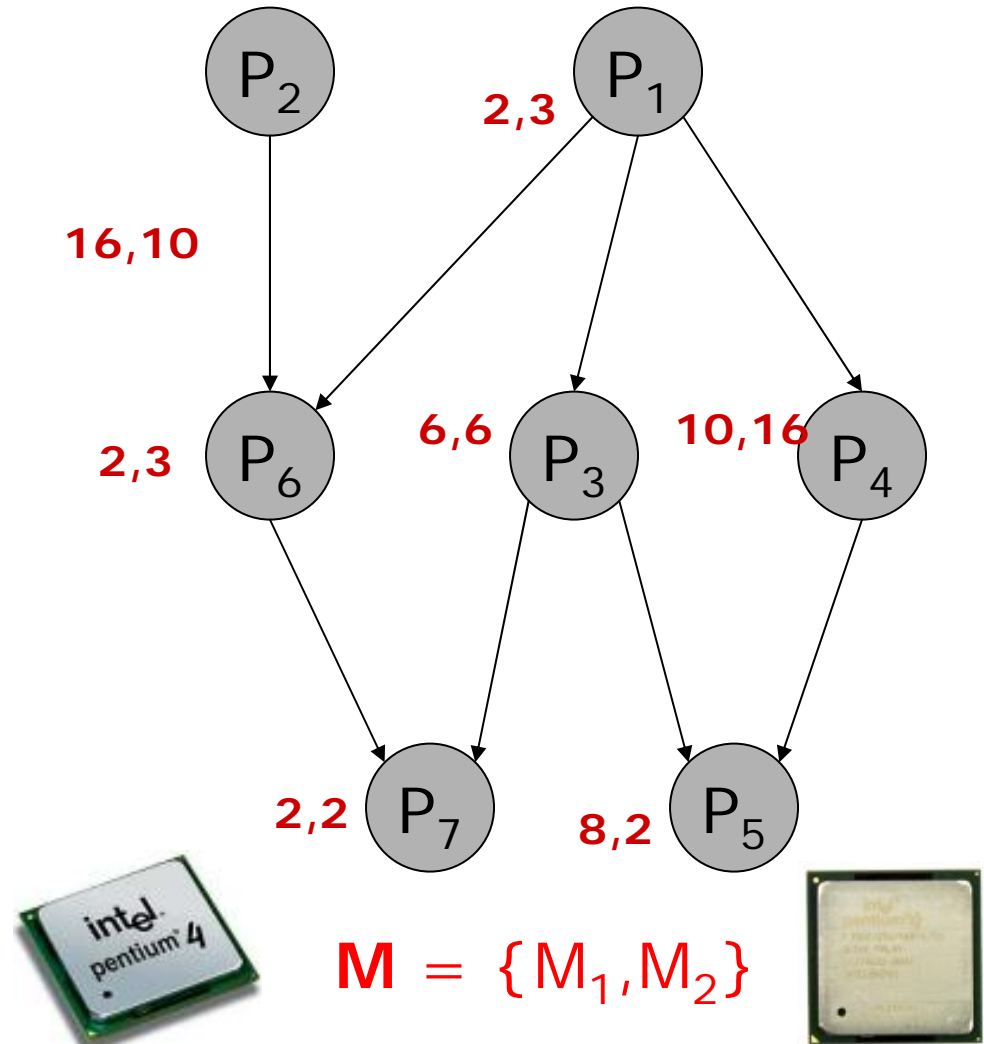
- ( Idle , Init , B=0 , x=0 )
- ( Idle , Init , B=0 , x=3.1415 ) d(3.1415)
- ( InUse , Using , B=6 , x=0 ) use
- ( InUse , Using , B=6 , x=6 ) d(6)
- ( Idle , Done , B=6 , x=6 ) done

# Task Graph Scheduling

## Optimal Static Task Scheduling



- Task  $\mathbf{P}=\{P_1, \dots, P_m\}$
- Machines  $\mathbf{M}=\{M_1, \dots, M_n\}$
- Duration  $\Delta : (\mathbf{P} \times \mathbf{M}) \rightarrow \mathbf{N}_\infty$
- $<$  : p.o. on  $\mathbf{P}$  (pred.)
  
- A task can be executed only if all predecessors have completed
- Each machine can process at most one task at a time
- Task cannot be preempted.
  
- Compute schedule with minimum completion-time!

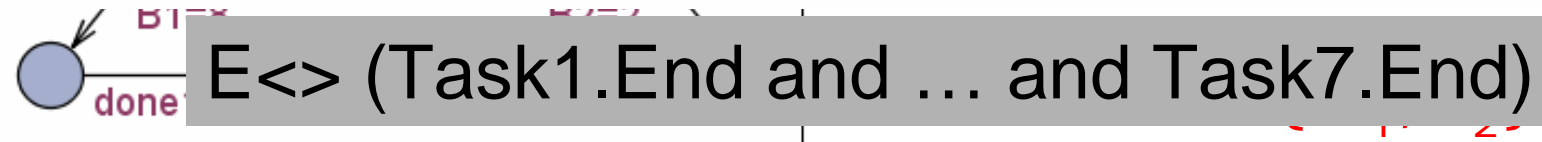
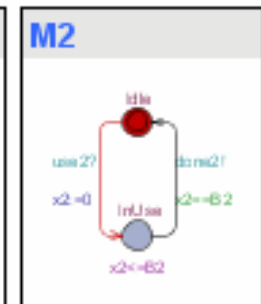
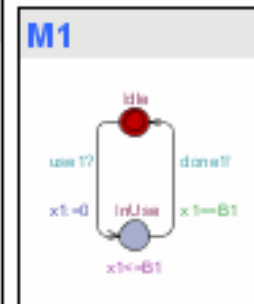
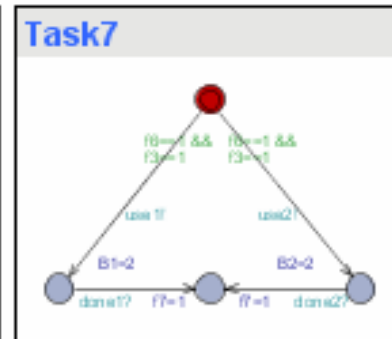
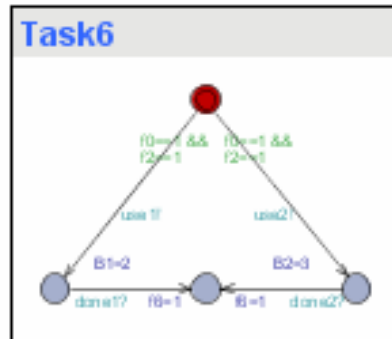
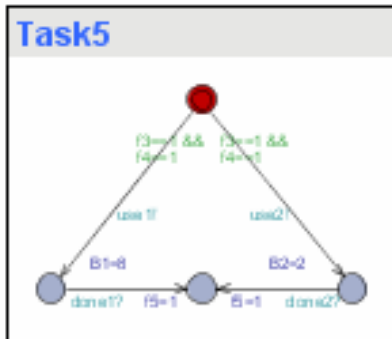
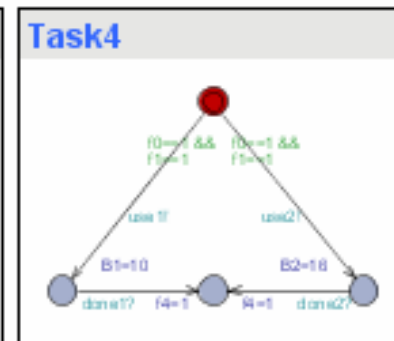
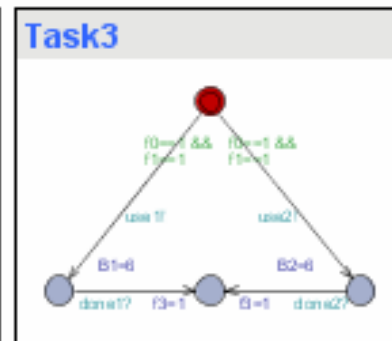
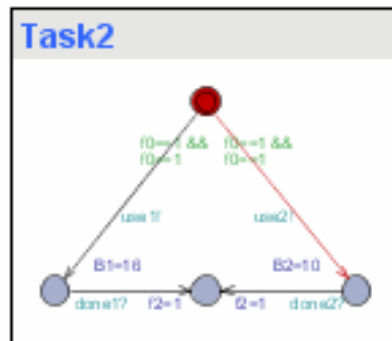
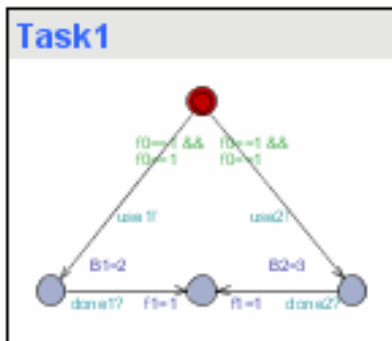
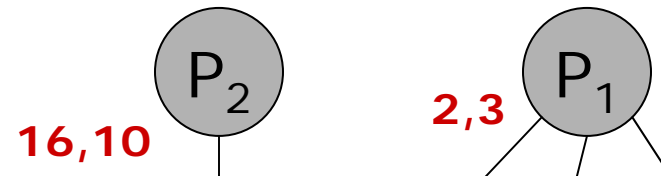


# Task Graph Scheduling

## Optimal Static Task Scheduling



- Task  $P = \{P_1, \dots, P_m\}$
- Machines  $M = \{M_1, \dots, M_n\}$





# Experimental Results



| name | #tasks | #chains | # machines | optimal | TA   |
|------|--------|---------|------------|---------|------|
| 001  | 437    | 125     | 4          | 1178    | 1182 |
| 000  | 452    | 43      | 20         | 537     | 537  |
| 018  | 730    | 175     | 10         | 700     | 704  |
| 074  | 1007   | 66      | 12         | 891     | 894  |
| 021  | 1145   | 88      | 20         | 605     | 612  |
| 228  | 1187   | 293     | 8          | 1570    | 1574 |
| 071  | 1193   | 124     | 20         | 629     | 634  |
| 271  | 1348   | 127     | 12         | 1163    | 1164 |
| 237  | 1566   | 152     | 12         | 1340    | 1342 |
| 231  | 1664   | 101     | 16         | t.o.    | 1137 |
| 235  | 1782   | 218     | 16         | t.o.    | 1150 |
| 233  | 1980   | 207     | 19         | 1118    | 1121 |
| 294  | 2014   | 141     | 17         | 1257    | 1261 |
| 295  | 2168   | 965     | 18         | 1318    | 1322 |
| 292  | 2333   | 318     | 3          | 8009    | 8009 |
| 298  | 2399   | 303     | 10         | 2471    | 2473 |



Symbolic A\*  
Brand-&-Bound  
60 sec

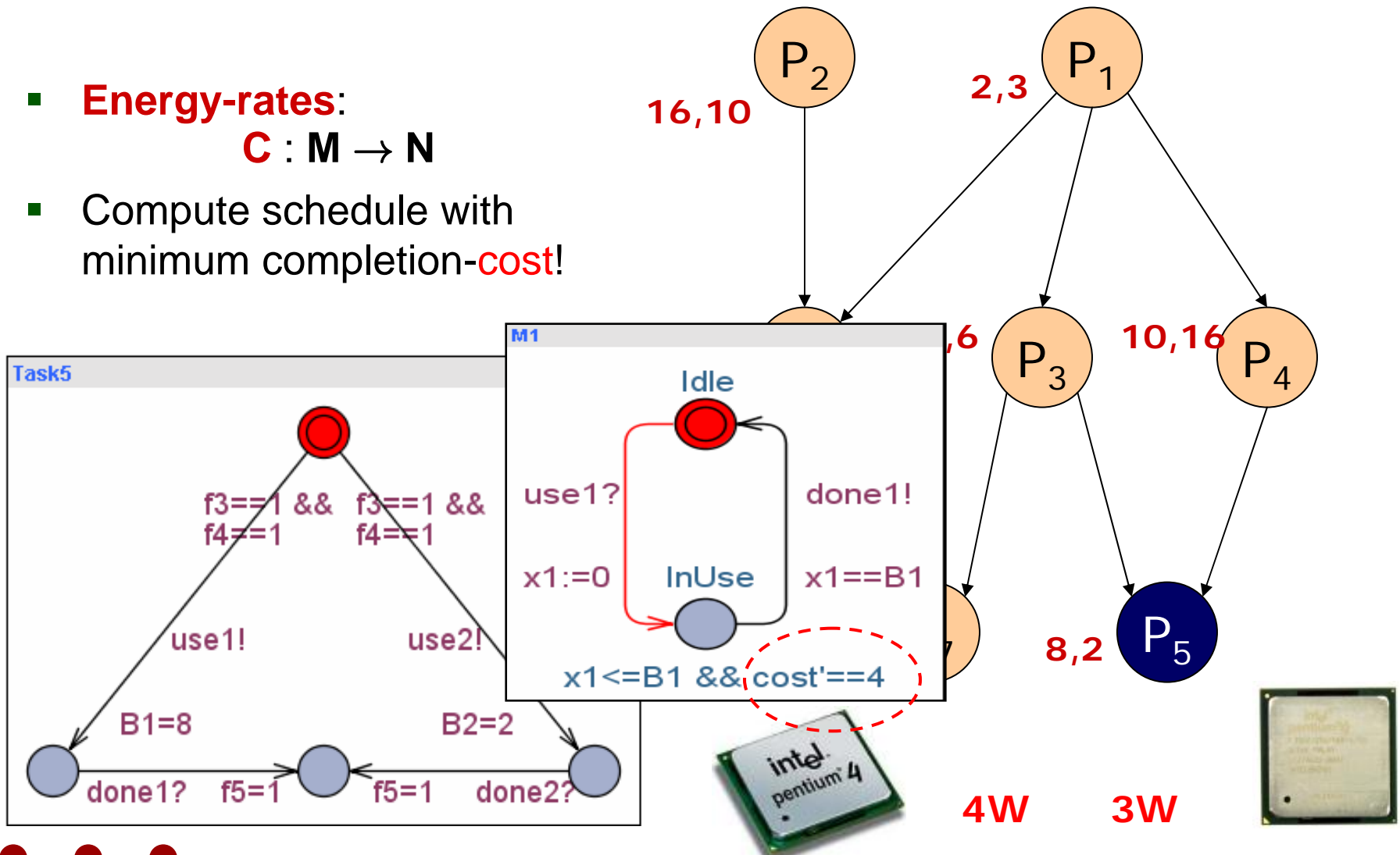
Abdeddaïm, Kerbaa, Maler

# Optimal Task Graph Scheduling

Power-Optimality



- Energy-rates:  
 $C : M \rightarrow N$
- Compute schedule with minimum completion-cost!



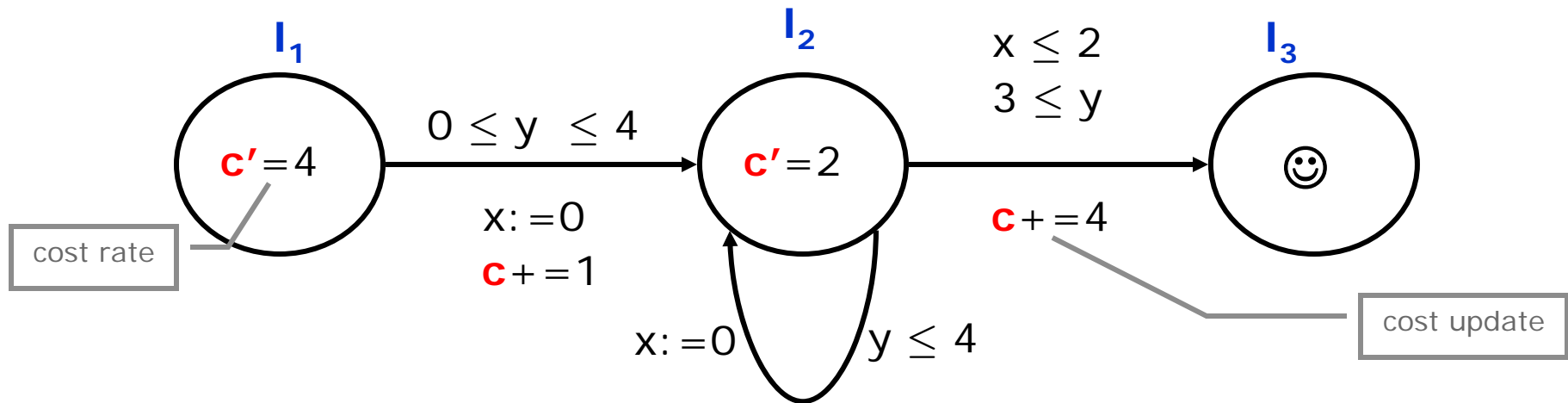
# Priced Timed Automata



Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

Timed Automata + **COST** variable



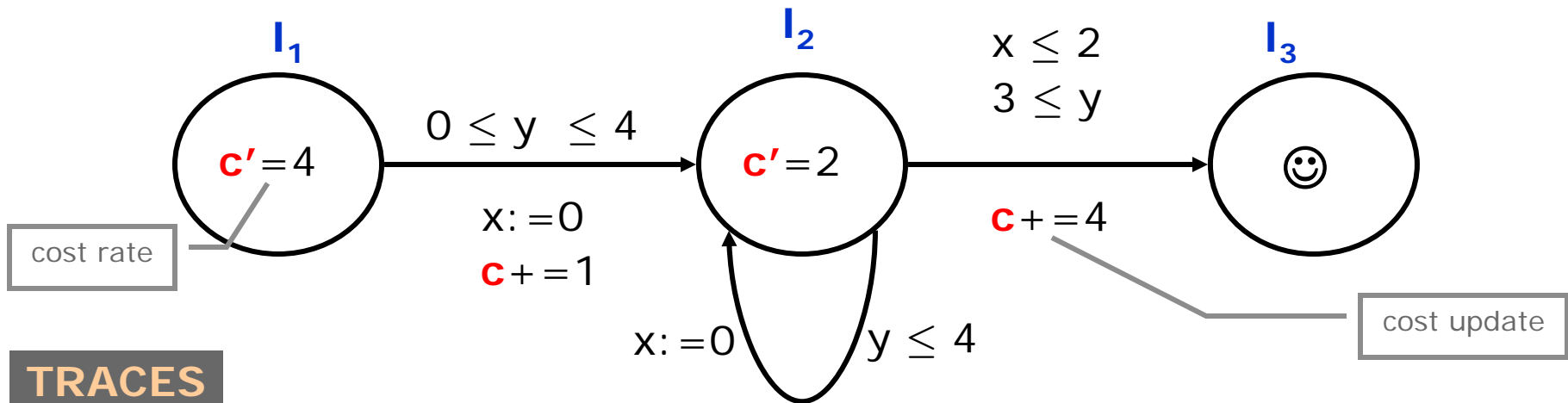
# Priced Timed Automata



Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

Timed Automata + **COST** variable



## TRACES

$$(l_1, x=y=0) \xrightarrow[12]{\varepsilon(3)} (l_1, x=y=3) \xrightarrow[1]{} (l_2, x=0, y=3) \xrightarrow[4]{} (l_3, \dots)$$

$$\sum c = 17$$



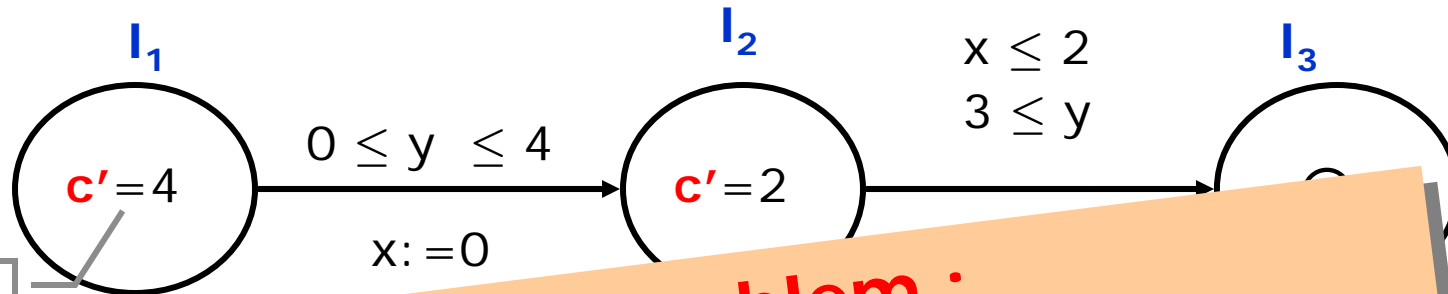
# Priced Timed Automata



Behrmann, Fehnker, et al (HSCC'01)

Alur, Torre, Pappas (HSCC'01)

Timed Automata + **COST** variable



## TRACES

$(l_1, x=y=0)$

$(l_1, x=y=0)^\varepsilon$

$(l_1, x=y=0) \xrightarrow{1} (l_2, x=0, y=0)$

**Problem :**  
Find the **minimum** (maximum) cost  
of reaching location  $l_3$

Efficient Implementation:  
CAV'01 and TACAS'04

Competitive with MILP  
and commercial tool (Axxon)

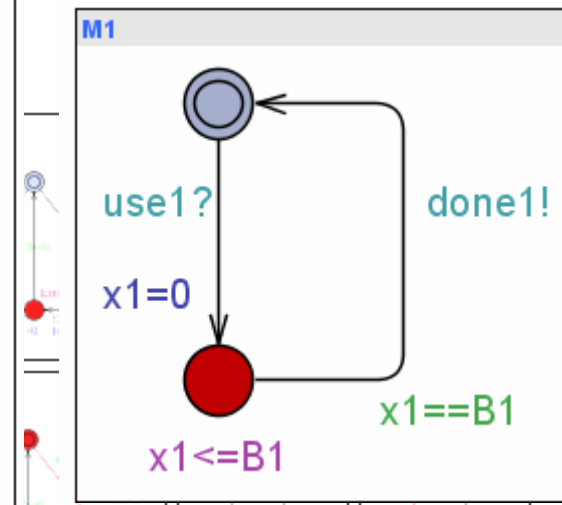
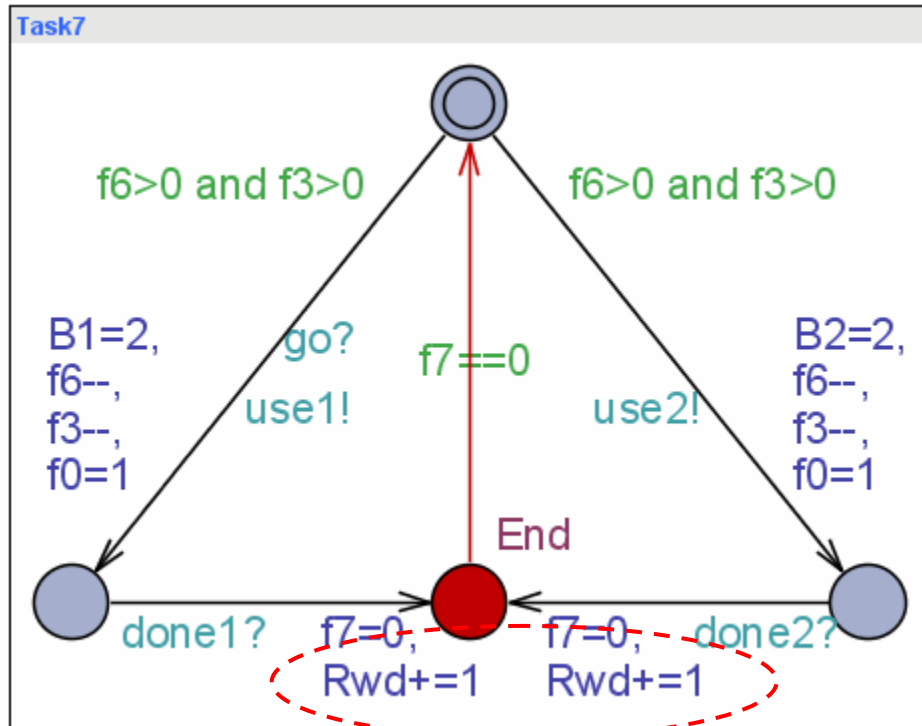
$\sum c = 17$

$(l_3, \_, \_)$   
 $\sum c = 16$

$(l_3, \_, \_)$   
 $\sum c = 11$



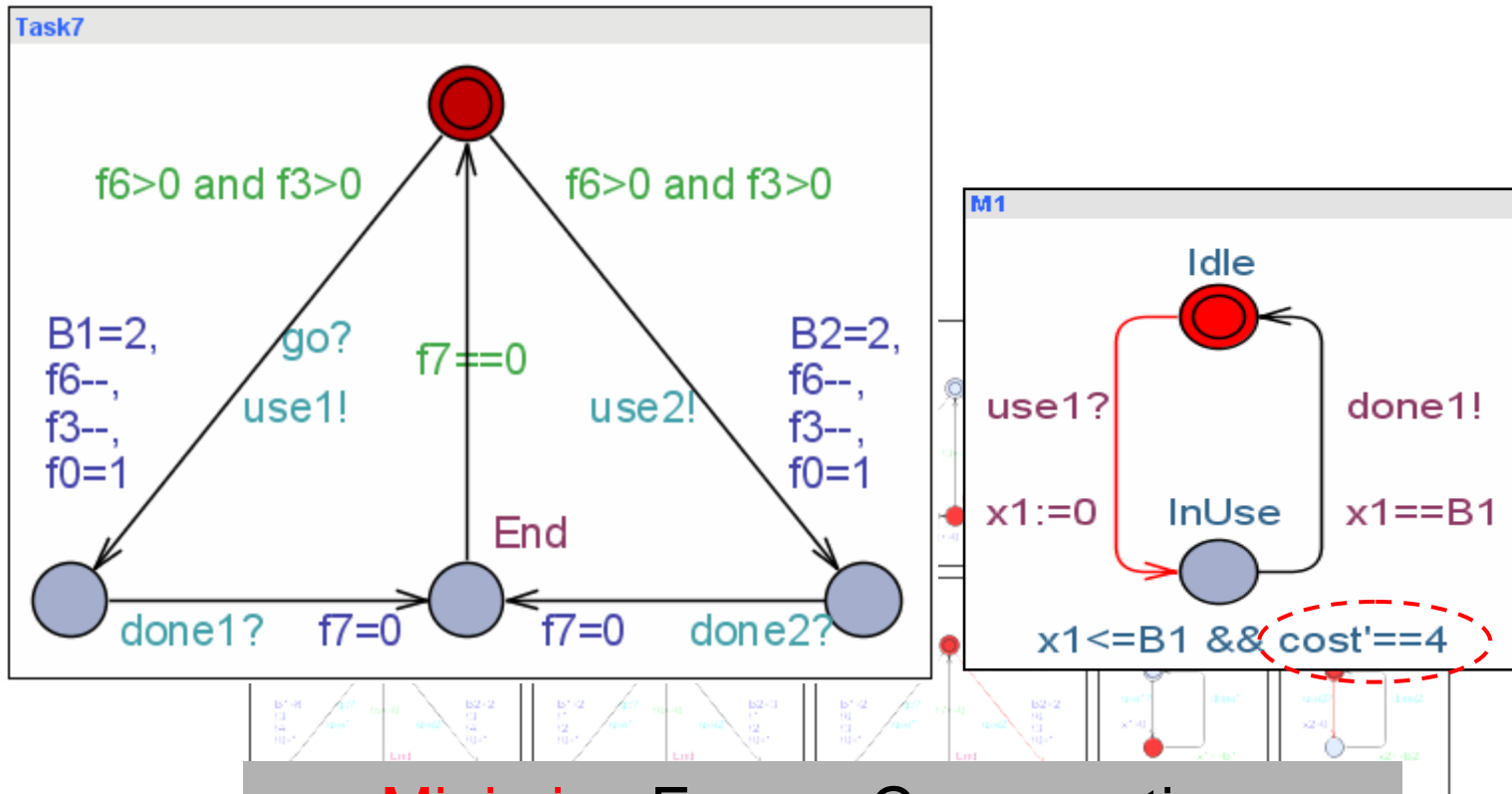
# Optimal Infinite Scheduling



Maximize throughput:  
i.e. maximize **Reward** / Time in the long run!



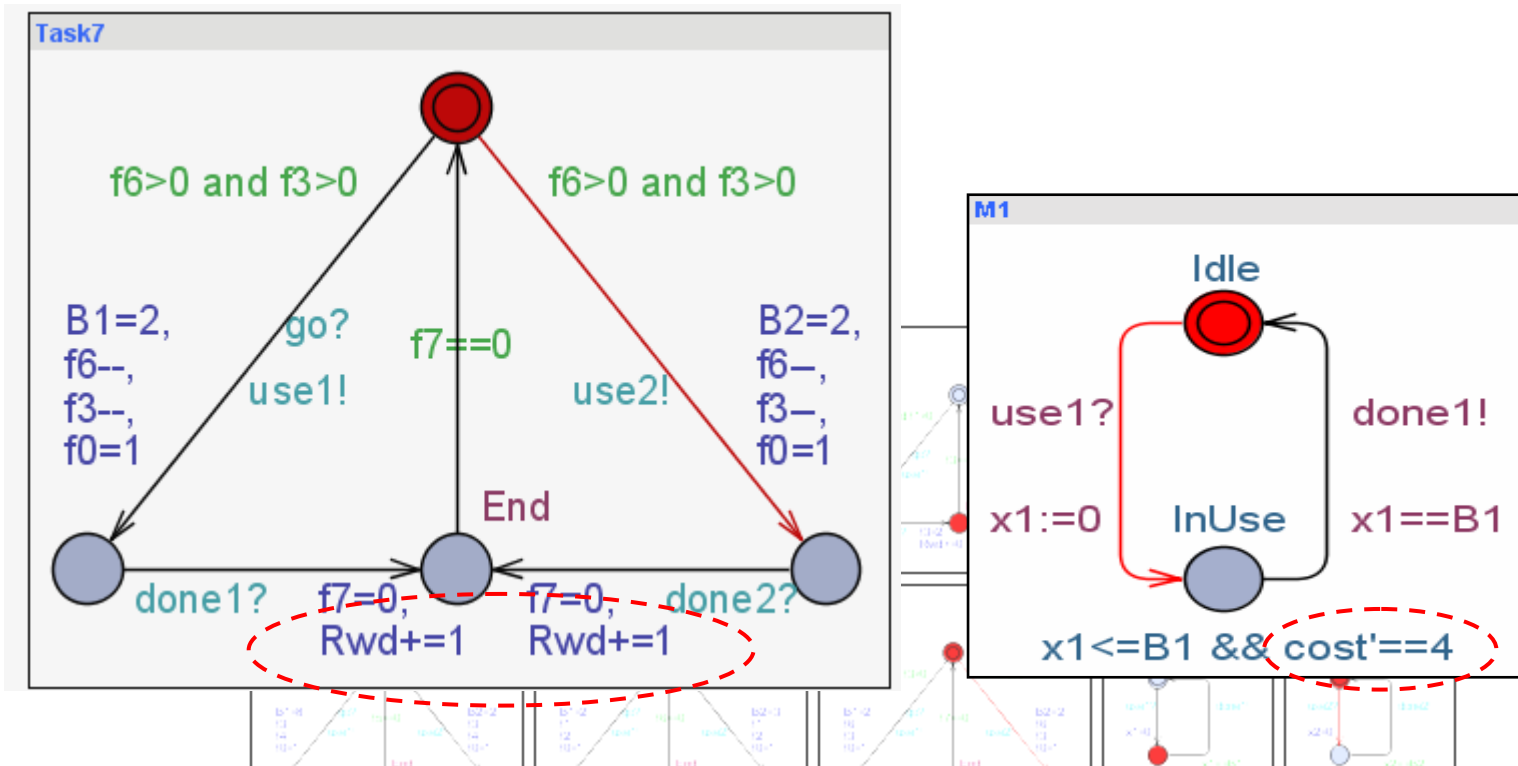
# Optimal Infinite Scheduling



**Minimize** Energy Consumption:  
i.e. minimize **Cost** / Time in the long run



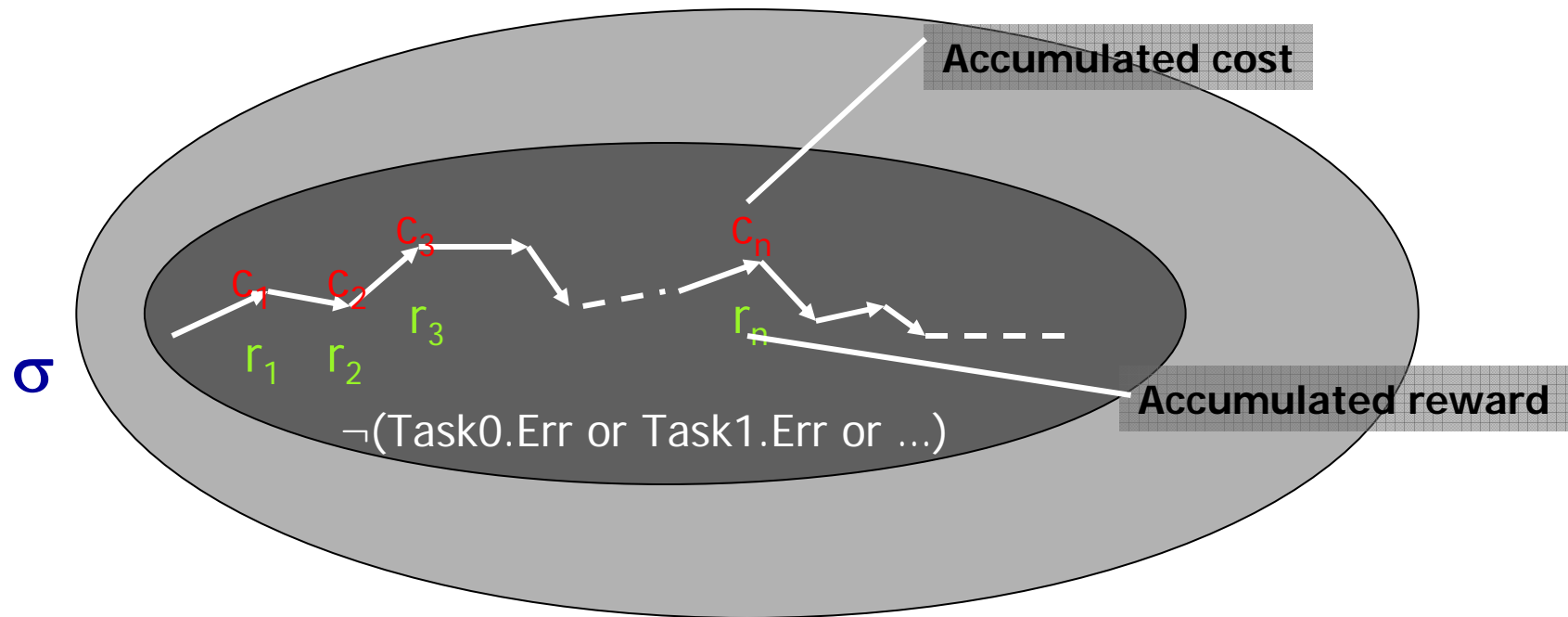
# Optimal Infinite Scheduling



Maximize throughput:  
i.e. maximize **Reward** / **Cost** in the long run



# Cost **Optimal** Scheduling = *Optimal Infinite Path*

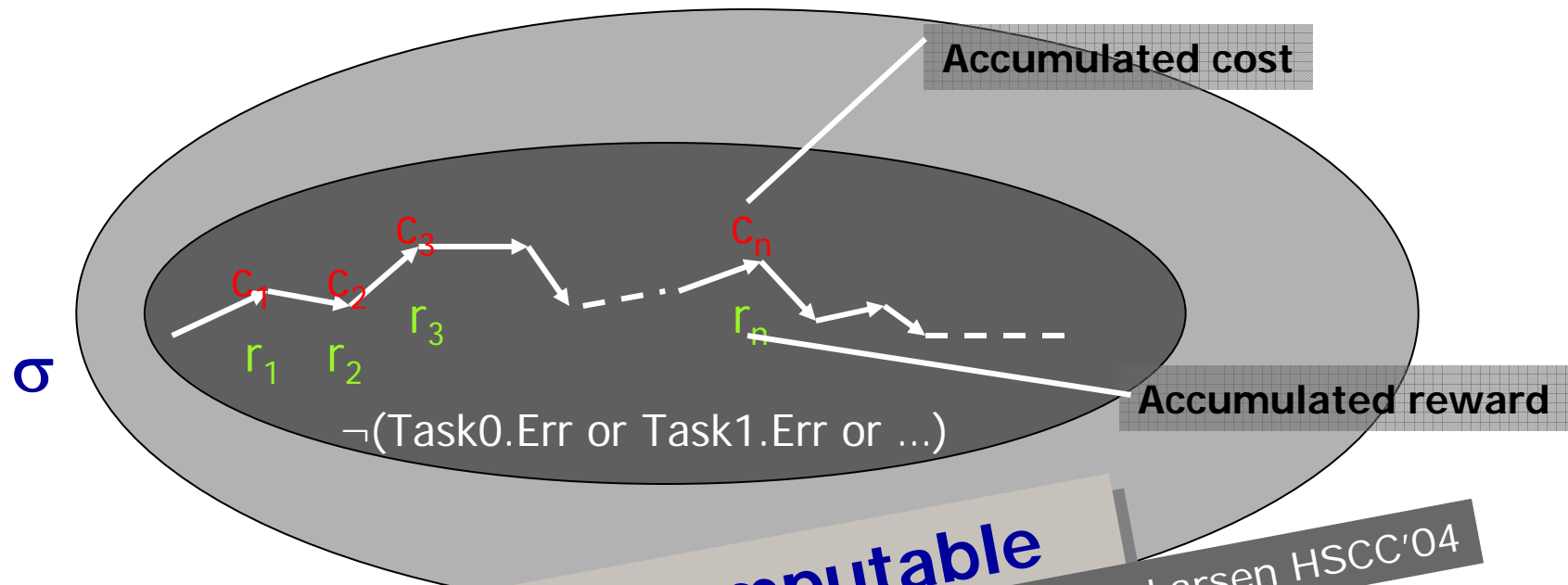


Value of path  $\sigma$ :  $\text{val}(\sigma) = \lim_{n \rightarrow \infty} c_n / r_n$

Optimal Schedule  $\sigma^*$ :  $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$



# Cost Optimal Scheduling = Optimal Infinite Path



**THEOREM:  $\sigma^*$  is computable**

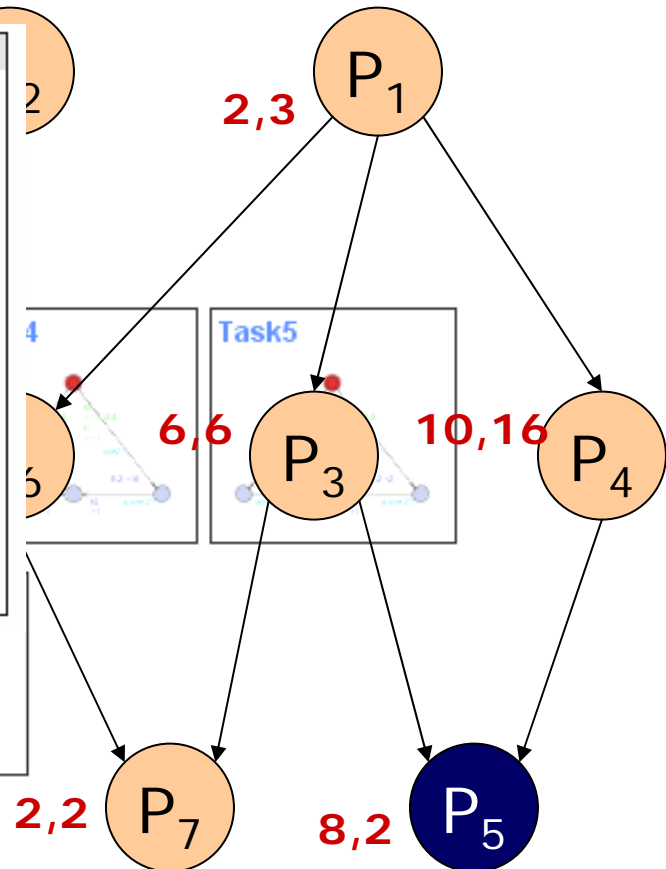
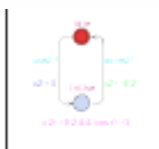
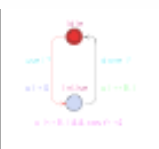
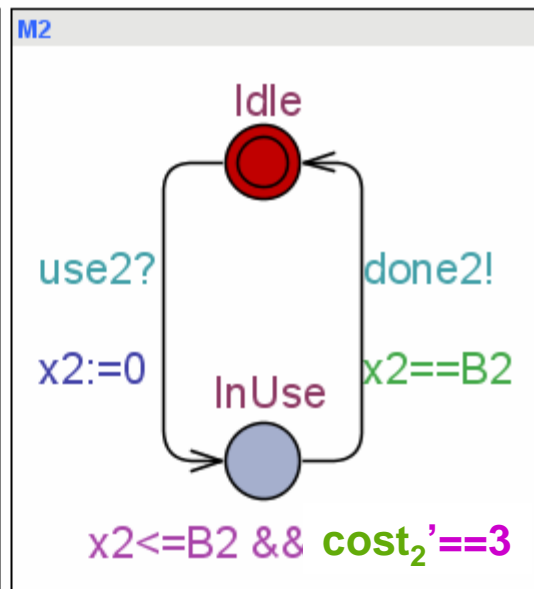
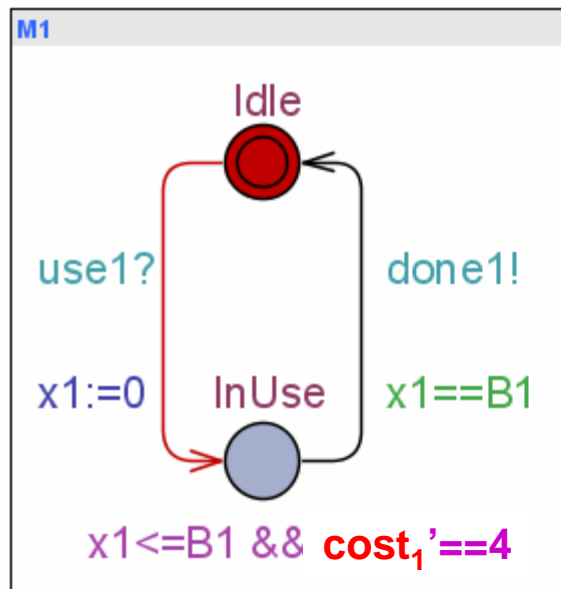
Bouyer, Brinksma, Larsen HSCC'04

For any path  $\sigma$ :  $\text{val}(\sigma) = \lim_{n \rightarrow \infty} c_n / r_n$

Optimal Schedule  $\sigma^*$ :  $\text{val}(\sigma^*) = \inf_{\sigma} \text{val}(\sigma)$



# Multiple Objective Scheduling

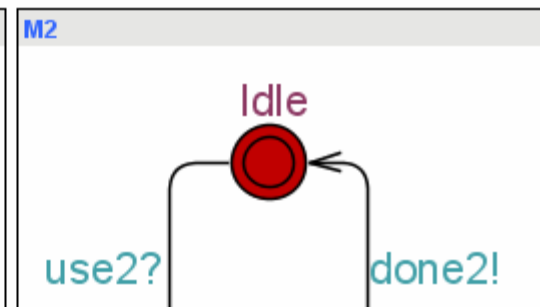
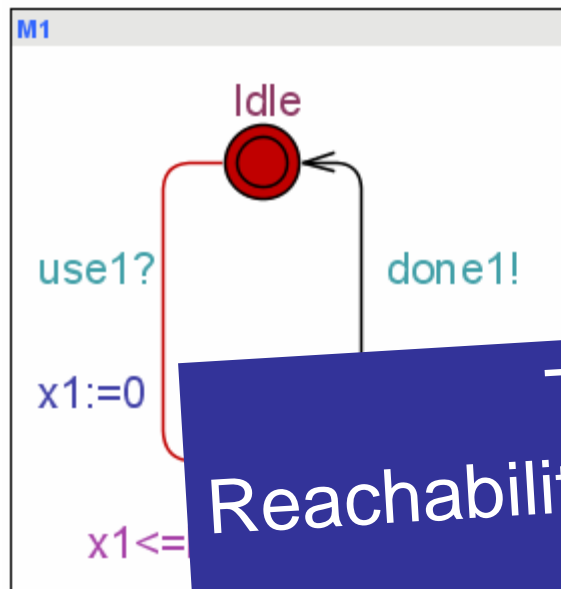


4W

3W



# Multiple Objective Scheduling

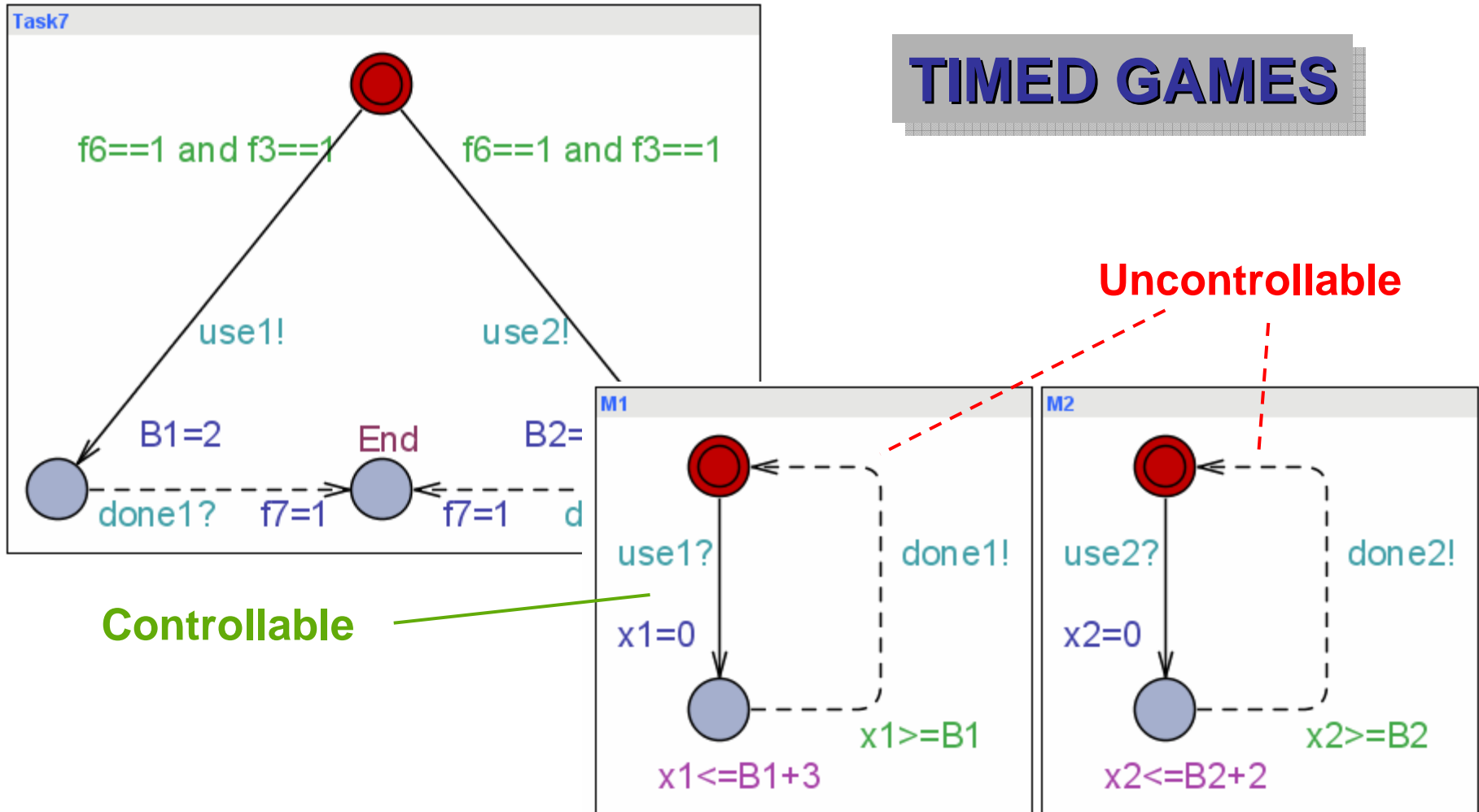


The **Pareto Frontier** for Reachability in Multi Priced Timed Automata is computable  
[Illum, Larsen FoSSaCS05]

# Synthesis = Scheduling under uncertainty



## TIMED GAMES

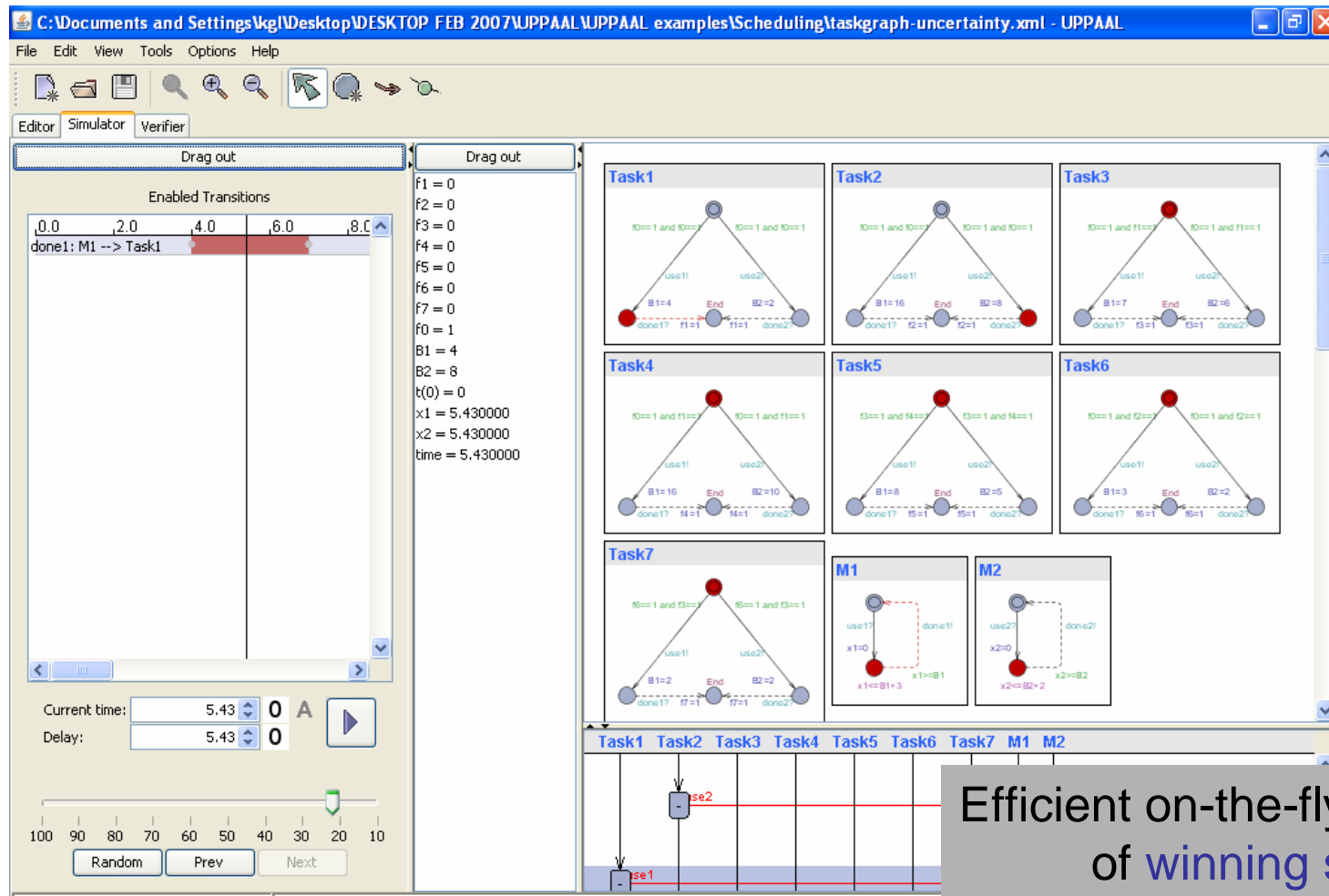


# UPPAAL Tiga

## Synthesis of winning strategies for *TIMED GAMES*

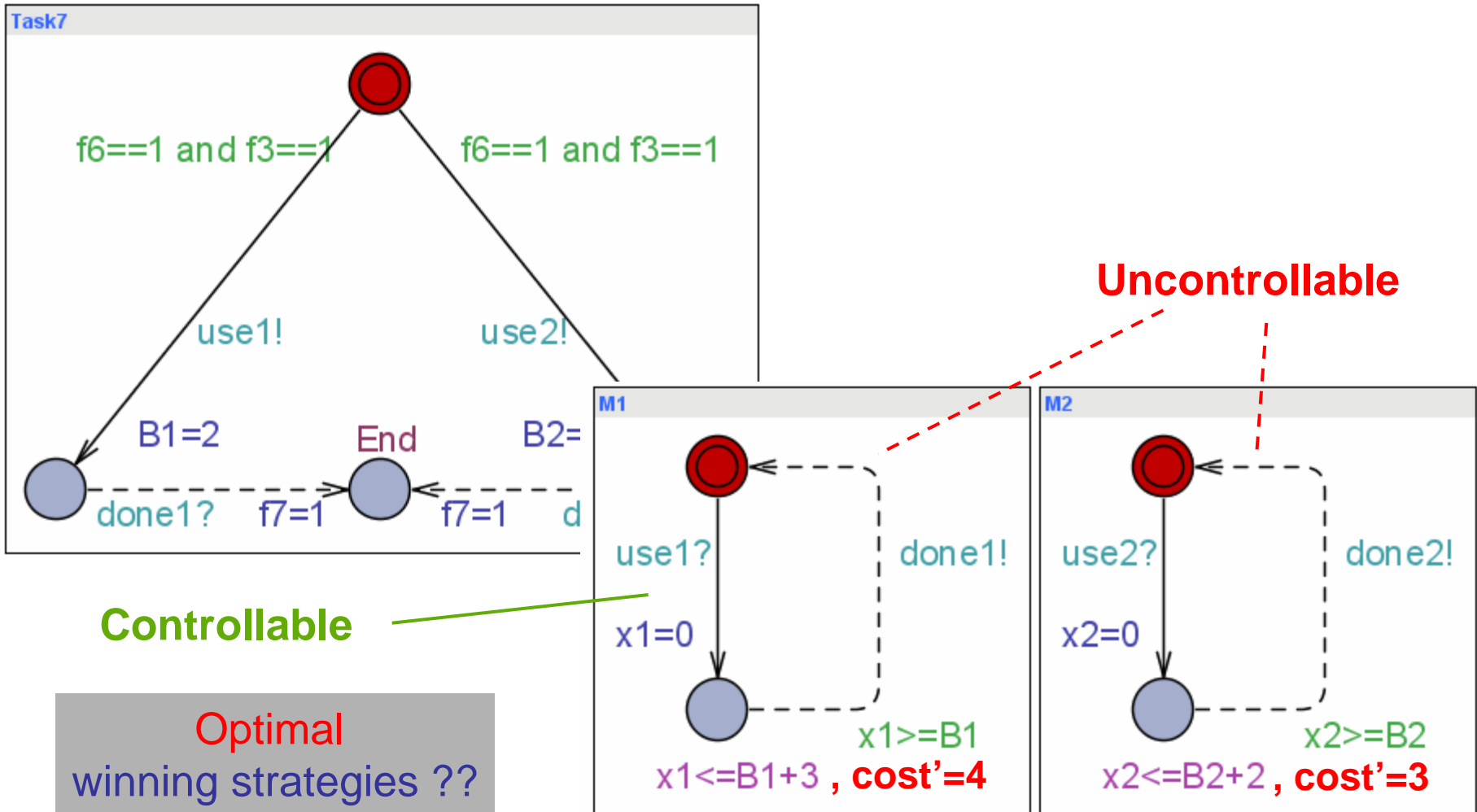


CONCUR05,  
CAV07,  
FORMATS07



Efficient on-the-fly generation  
of winning strategies for  
safety & liveness objectives

# Optimal Synthesis = Priced Timed Games



# Priced Timed Games



- Price Optimal Control (reachability):
  - Acyclic PTA [LTMM02]
  - Bounded length [ABM04]
  - Strong non-zero cost-behaviour [BCFL04]
  - **Undecidable** with 3 clocks or more [BBR05, BBM06]
  - **Decidable** for PTGs with 1 clock [BLMR06]
- Priced Timed **Safety** Games
  - Conjectured to be undecidable in general.



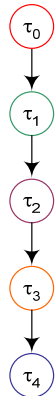
# Handling realistic applications?



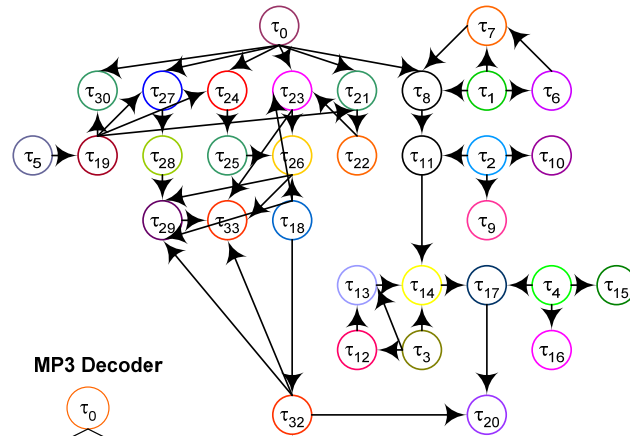
## Smart phone:



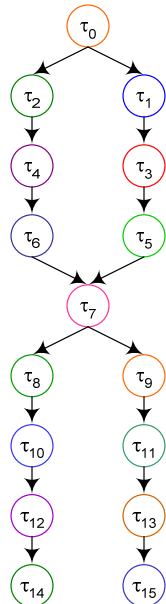
JPEG Encoder



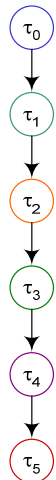
GSM Decoder



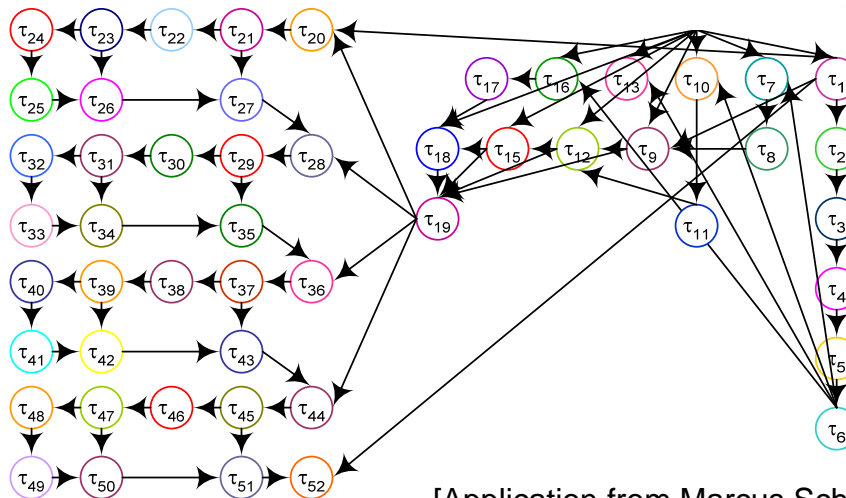
MP3 Decoder



JPEG Decoder

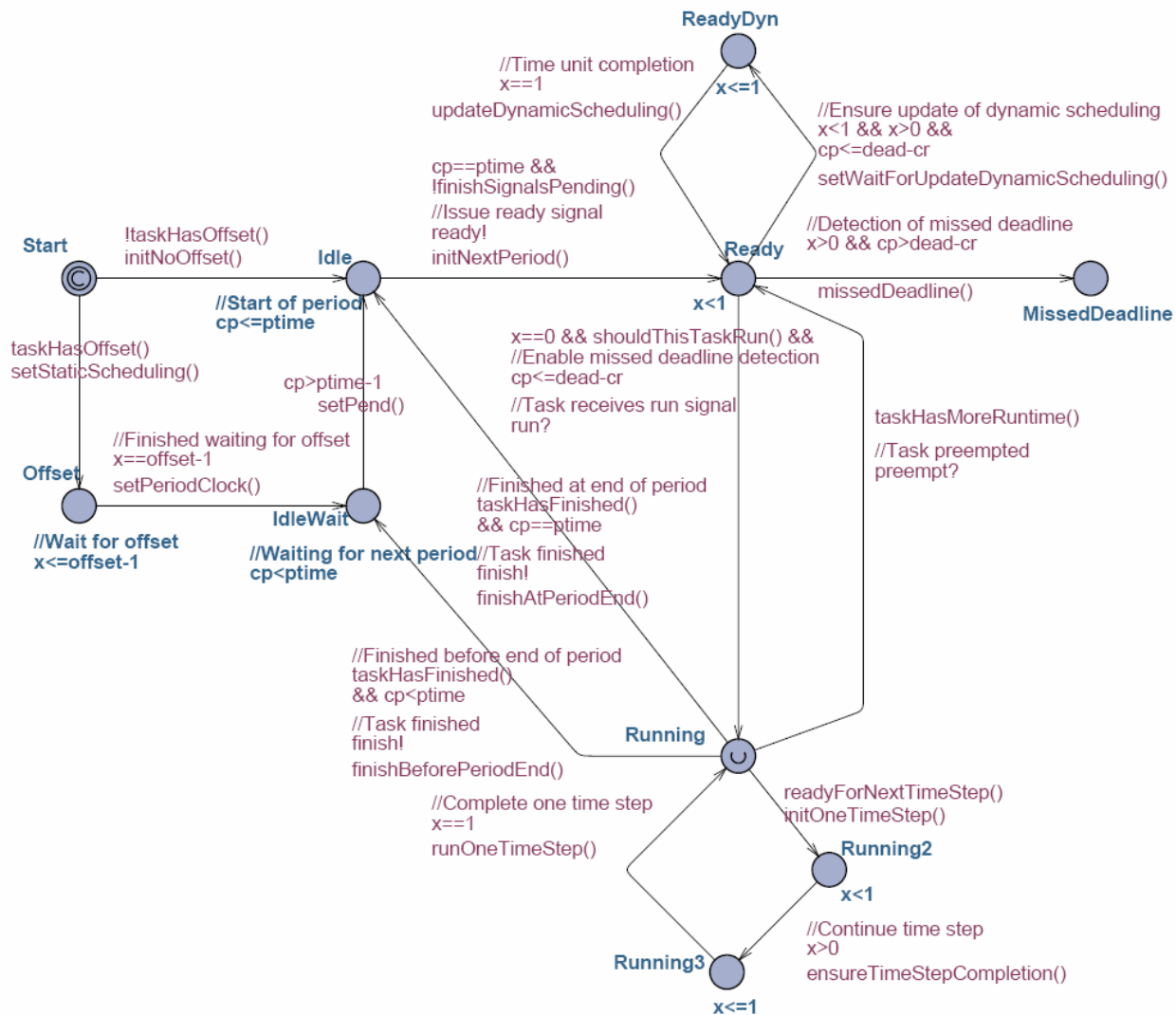


GSM Encoder



[Application from Marcus Schmitz, TU Linkoping]

# Timed Automata for a task

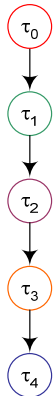




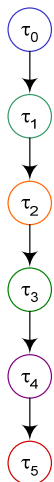
# Smart phone



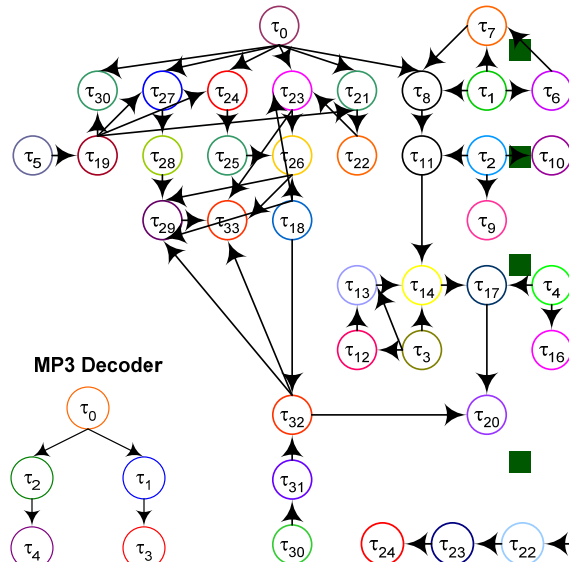
JPEG Encoder



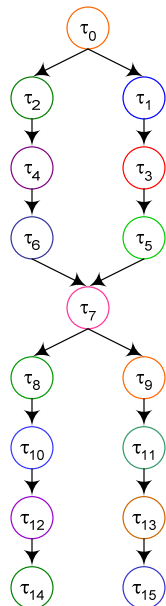
JPEG Decoder



GSM Decoder



MP3 Decoder



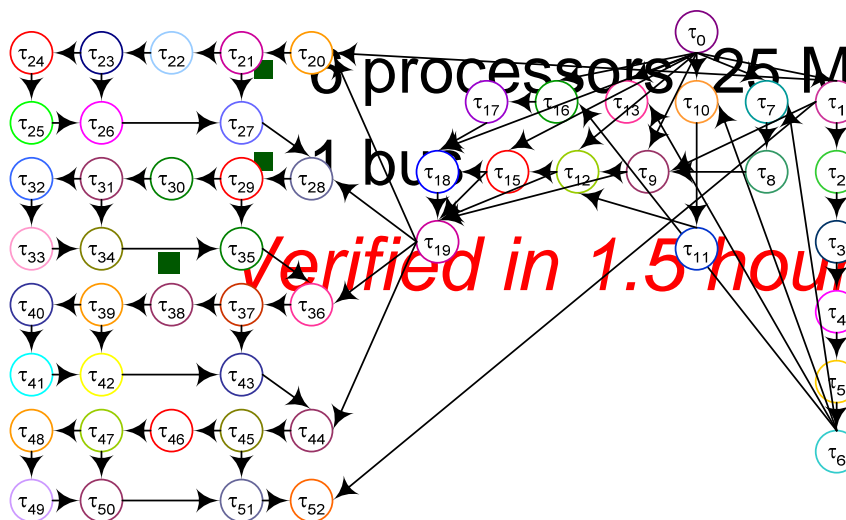
Tasks: 114

Deadlines: [0.02: 0.5] sec

Execution: [52 : 266.687] cycles

Platform:

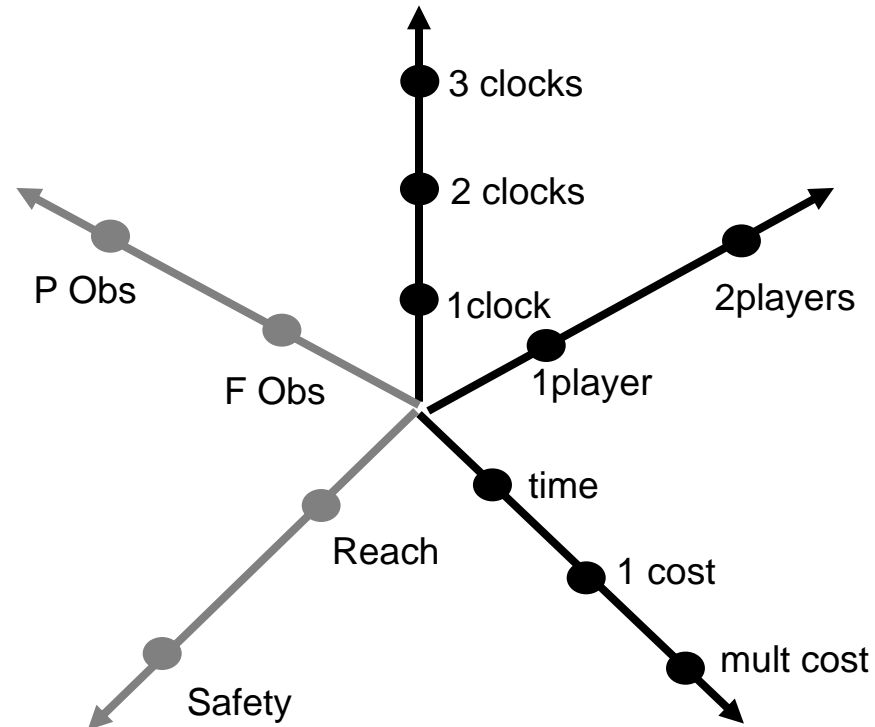
3 processors, 25 MHz bus



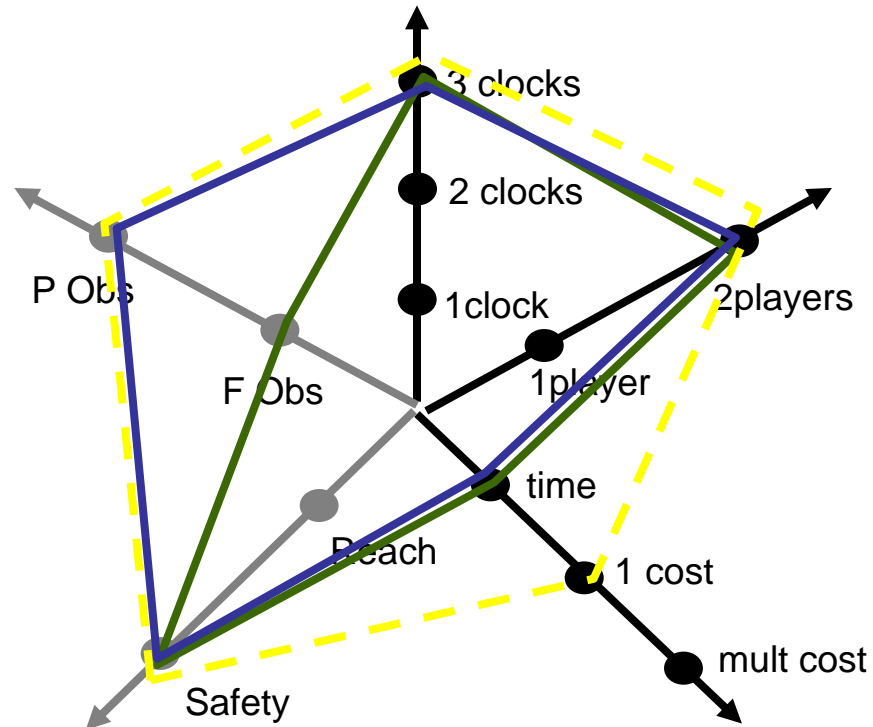
*Verified in 1.5 hours!*



# Optimal Combinations



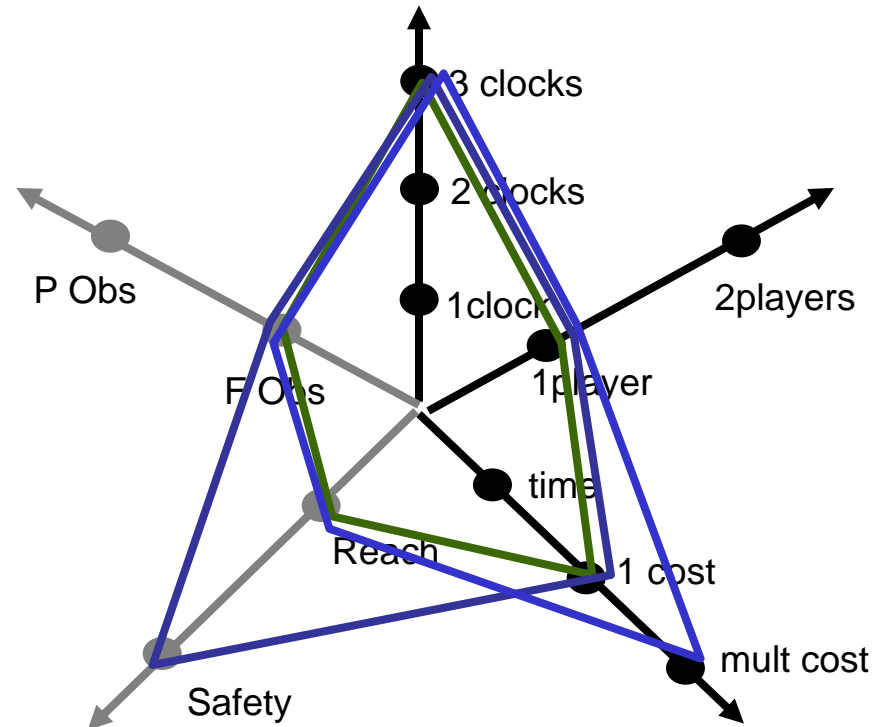
# Optimal Combinations



**TIGA**



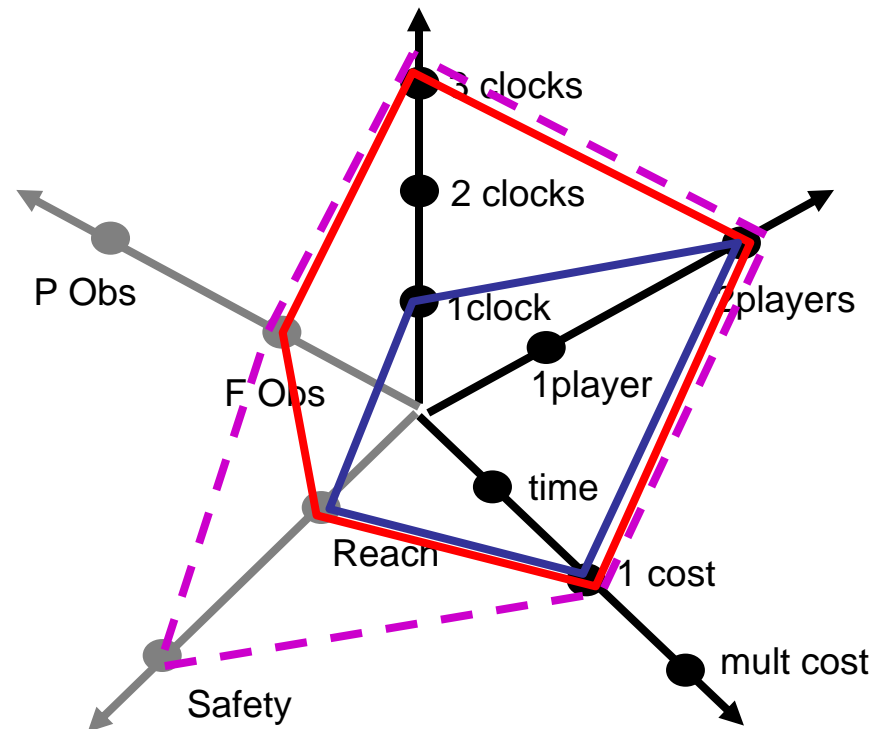
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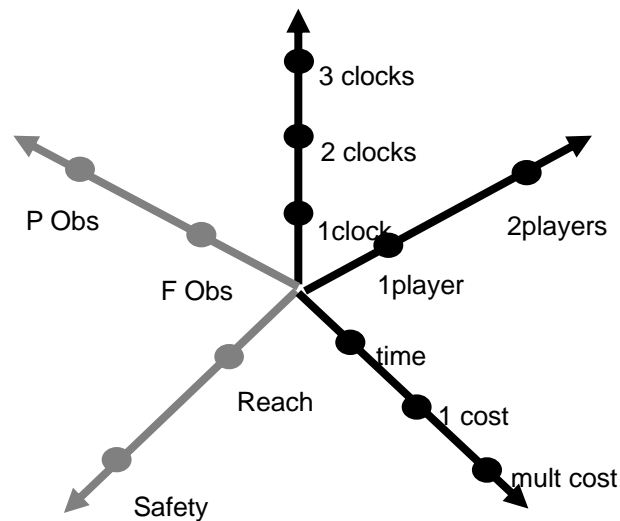
**CORA**



# Optimal Combinations



# Conclusion



- Identification of all Pareto optimal combinations!
  - Safety for PTG?
  - Reachability for 2PTG?
  - Safety for MPTA?
  - Safety for 1PTG?
- Efficient realizations
  - TG w PO?
  - Safety for PTA?
  - Reachability for 1PTG?
- Dealing with undecidability.
- $\forall \epsilon. |\text{Th-Prac}| < \epsilon$





Thanks for your  
attention!

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