Does Secure Time-Stamping Imply Collision-Free Hash Functions?

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Outline

1. Security of hash functions and why this is important
2. Timestamping and backdating attack
3. Blackbox reductions
4. Timestamping doesn't require CRHF

Hash functions

- $X \in \{0,1\}^*$, $x = h(X)$, $x \in \{0,1\}^m$
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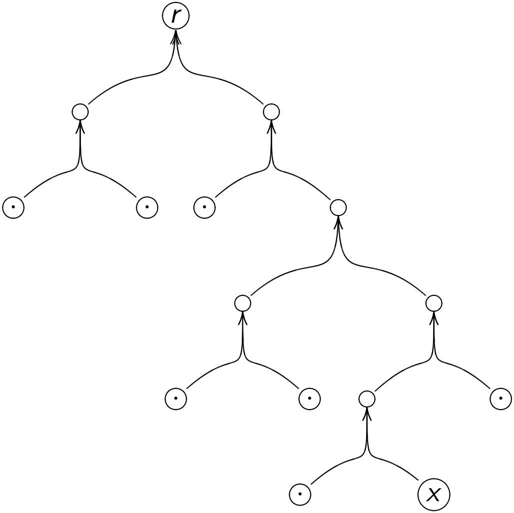
- Recent attacks against collision resistance of MD5, SHA-0, SHA-1.
- Is this *collision freedom* really required in applications?
- For example, in time-stamping:
  - Buldas and Saarepera in 2004: collision freedom is *insufficient* for security of timestamping.
  - Buldas and Laur in 2006: collision freedom is *unneccessary* for security of timestamping.
  - This paper: secure time-stamping schemes may exist without CRHF.

Timestamping

- ... is about vouching the existence of some value x at a certain time.
- Works in batches $\mathcal{X}_1, \mathcal{X}_2, \ldots$, where $\mathcal{X} = \{x_1, x_2, \ldots, x_m\}$.
- TS service provider publishes *commitments* $r = \mathrm{Com}(\mathcal{X})$ after each round.
- When certain value is timestamped, a *certificate* $c = \mathrm{Cert}(\mathcal{X}, x)$ is computed.
- To verify the certificate, there is a function $\mathrm{Ver}(r, x, c) = \mathrm{yes}$ in case everything is correct.

Hash-tree time-stamping certificate chain



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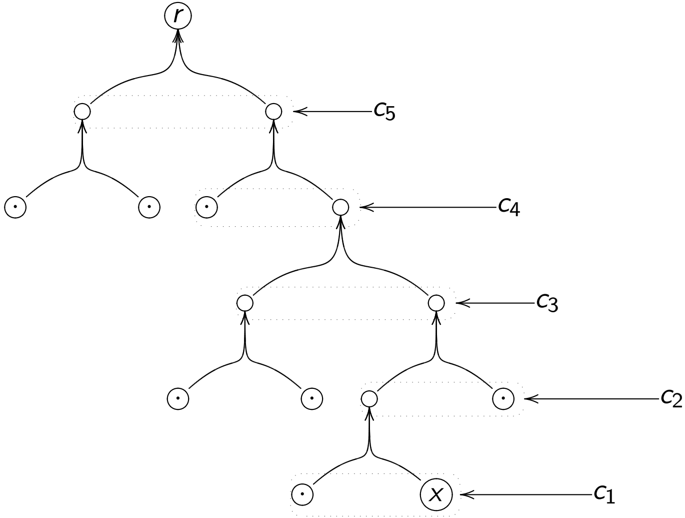
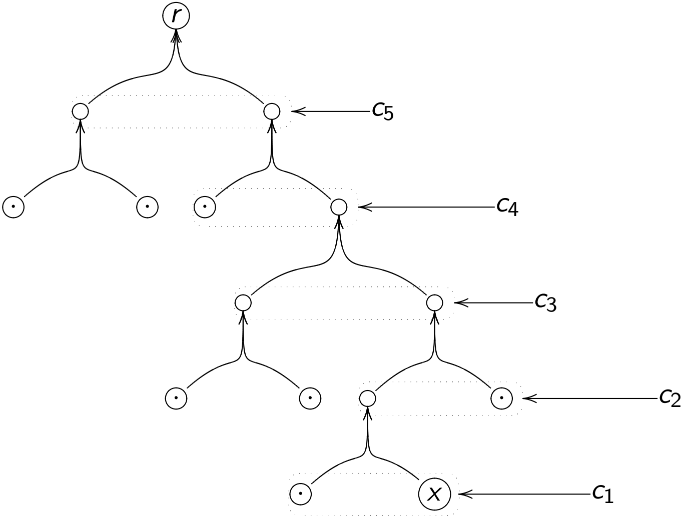


Hash-tree time-stamping certificate chain

$c = (c_1, \ldots, c_5), r = h(c_5), c_5 = h(c_4), c_4 = h(c_3), c_3 = h(c_2), c_2 = h(c_1)$



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- $x = H(\mathcal{D}'_\mathrm{A})$
- $c = \mathsf{Cert}(\mathcal{X}, x)$
- $\mathsf{Ver}(r, x, c) = \mathsf{yes}$

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- $\mathsf{A} = (\mathsf{A}_1, \mathsf{A}_2) \in \mathsf{FPU}$, i.e.

$$\Pr\Big[(r,a) \leftarrow \mathsf{A}_1(1^k), x' \leftarrow \Pi(r,a), (x,c) \leftarrow \mathsf{A}_2(r,a) : x' = x\Big] = k^{-\omega(1)}$$

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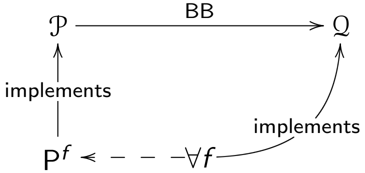

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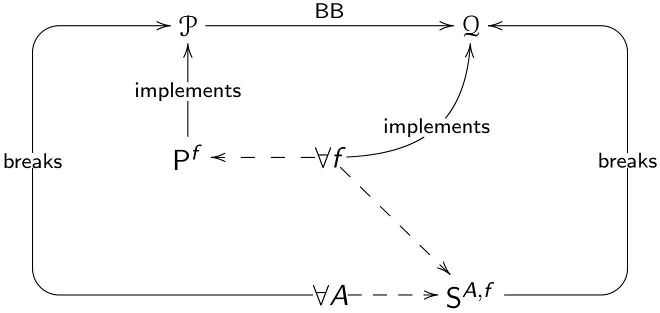

- Construction condition:
  $\forall f$ (any function) implementing $\mathcal{Q}$, P^f implements $\mathcal{P}$,
- Guarantee condition:
  $\forall A \forall f$ (any function) if A breaks P^f (as $\mathcal{P}$), then $\mathsf{S}^{A,f}$ breaks f (as $\mathcal{Q}$).

Fully blackbox reduction. Examples

- PRNG blackbox reduction to one-way functions

PRNG	$\mathcal{P} \xrightarrow{\text{BB}} \mathcal{Q}$		*OWF*

- Bounded time-stamping scheme blackbox reduction to CRHF

TS	$\mathcal{P} \xrightarrow{\text{BB}} \mathcal{Q}$		*CRHF*

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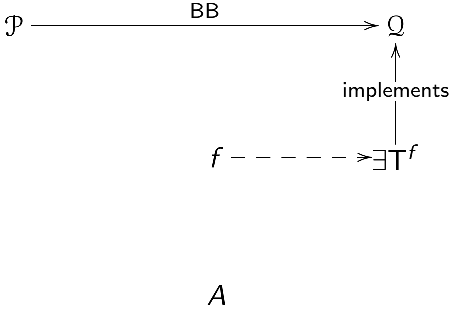

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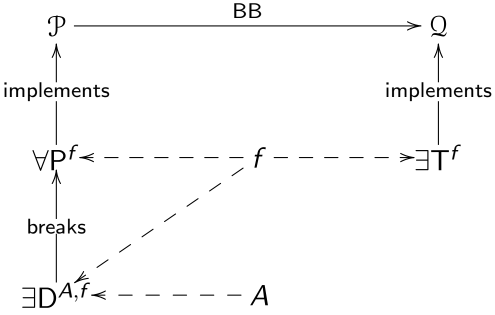

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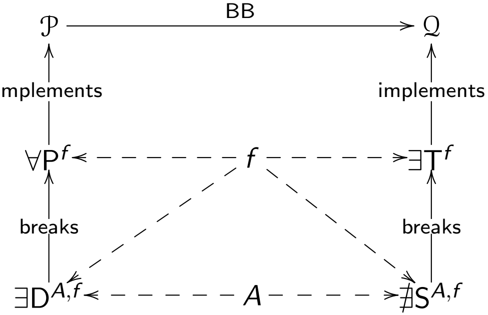

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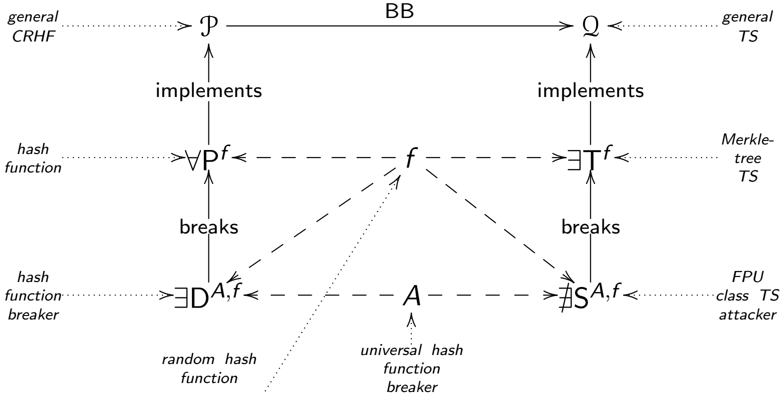
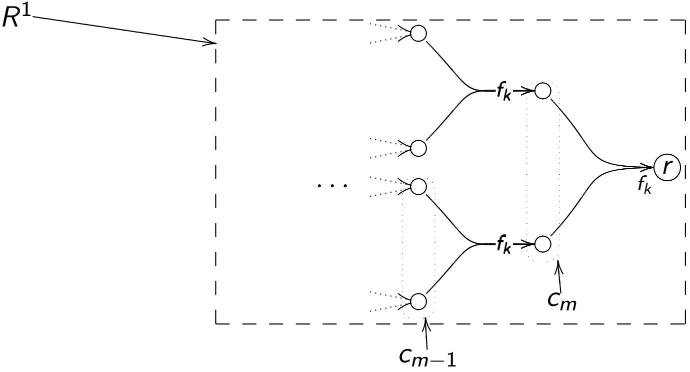


Connecting the hash chain $c = (c_1, c_2, \ldots, c_{m-1}, c_m)$



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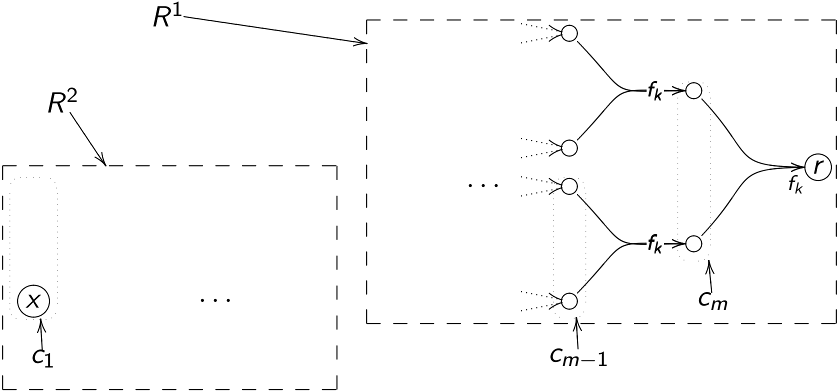

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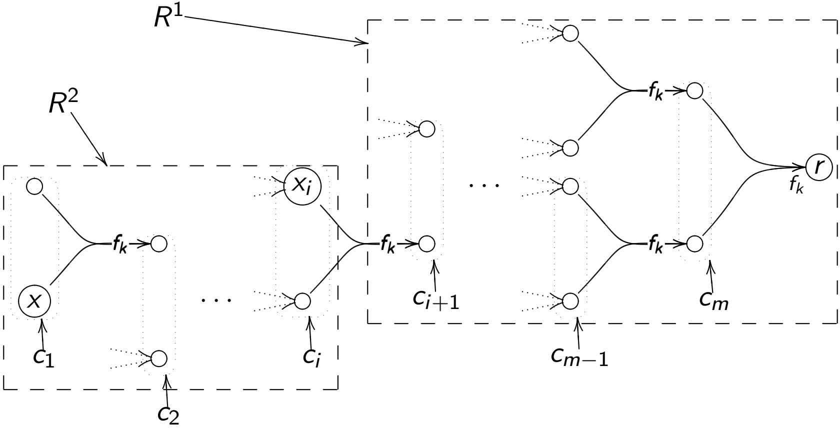

Probability of getting a hit

- A is a universal hash function collision finder: $(x, x') \leftarrow A(''F'')$ and $F(x) = F(x')$
- A works by:
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- probability of getting a collision is high, but probability of getting a *hit* is still low
- therefore the powerful A is not particularly useful in our case.

Conclusions

$$\Pr\Big[(r,a) \leftarrow \mathsf{S}_1^{A,f}(1^k), (x,c) \leftarrow \mathsf{S}_2^{A,f}(r,a) : \\ \mathsf{Ver}(x,c,r) = \mathsf{yes}\Big] = k^{-\omega(1)}$$

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- Blackbox reduction of CRHF to TS is not possible.
- In blackbox sense TS $\not\Rightarrow$ CRHF.
- Secure timestamping may exist even if there are no collision-resistant hash functions.