Generic proofs and programs for coinductive types in Martin-Löf Type Theory (short abstract)

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Coinductive types and coinductive reasoning are important tools in study of reactive systems. In the context of dependent types and proof assistants, they have been studied amongst others by Gimenéz [Gim96]. In our work, we analyze possibility of applying the "generic proofs" approach formulated in [BDJ03] in this area. Here we want to present some preliminary results in this field. Roughly speaking, the approach to generic programming in [BDJ03] is for a given class of types to define a corresponding universe, i.e. a set of codes Sig and a decoding function

$$\mathrm{T} : \mathrm{Sig} \to \mathrm{Set}$$

Then a generic iterator is defined basing on the initial algebra diagram:

$$\begin{tikzcd}
\mathrm{F}_\Sigma \mathrm{T}_\Sigma \arrow[r, "\mathrm{Intro}_\Sigma"] \arrow[d, "\mathrm{F}_\Sigma(\mathrm{iter}_\Sigma\, C\, d)"'] & \mathrm{T}_\Sigma \arrow[d, "\mathrm{iter}_\Sigma\, C\, d"] \\
\mathrm{F}_\Sigma\, C \arrow[r, "d"'] & C
\end{tikzcd}$$

For most universes, instead of initial algebras and iterators, one can consider final coalgebras and coiterators

$$\begin{tikzcd}
\mathrm{F}_\Sigma \mathrm{T}_\Sigma & \mathrm{T}_\Sigma \arrow[l, "out_\Sigma"'] \\
\mathrm{F}_\Sigma\, C \arrow[u, "\mathrm{F}_\Sigma(\mathrm{coiter}_\Sigma\, C\, d)"] & C \arrow[l, "d"] \arrow[u, "\mathrm{coiter}_\Sigma\, C\, d"']
\end{tikzcd}$$

(even though it is slightly more complicated than just reversing all arrows).

We define a universe for (co)inductive families and show how it can be used todefine generic equality predicate for coinductive types. Furthermore, we generalize ideas of Gimenéz [Gim] by defining notion of bisimulation and Park's principle of co-induction a generic way.

References

[BDJ03] Marcin Benke, Peter Dybjer, and Patrik Jansson. Universes for generic programs and proofs in dependent type theory. *Nordic Journal of Computing*, 10(4):265–289, 2003.

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