Extracting a Data Flow Analyser in Constructive Logic

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Static program analysis

The goals of static program analysis

- To prove properties about the run-time behaviour of a program
- In a fully automatic way
- Without actually executing this program

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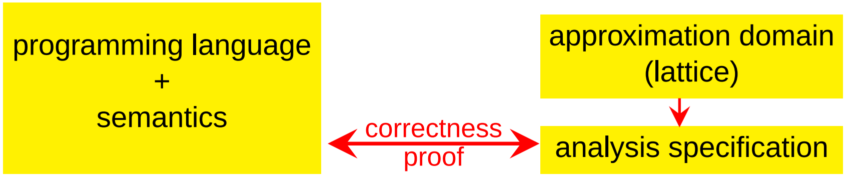
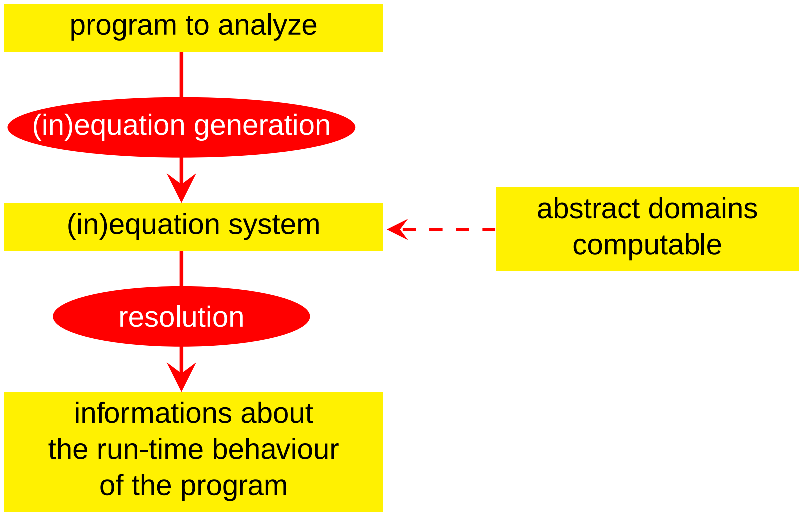
Solid foundations for designing an analyser

- Formalization and correctness proof by abstract interpretation
- Resolution of constraints on lattices by iteration and symbolic computation

Formalization



Resolution


program to analyze
(in)equation generation
(in)equation system
abstract domains computable
resolution
informations about the run-time behaviour of the program

So what’s the problem ?

Formalization part

$$\ddot{\alpha}\llbracket P\rrbracket(\mathrm{Post}\llbracket \mathtt{if}\ B\ \mathtt{then}\ S_t\ \mathtt{else}\ S_f\ \mathtt{fi}\rrbracket)$$

$=$ $\wr$def. (110) of $\ddot{\alpha}\llbracket P\rrbracket\wr$

$$\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{Post}\llbracket \mathtt{if}\ B\ \mathtt{then}\ S_t\ \mathtt{else}\ S_f\ \mathtt{fi}\rrbracket \circ \ddot{\gamma}\llbracket P\rrbracket$$

$=$ $\wr$def. (103) of Post$\wr$

$$\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{post}[\tau^\star\llbracket \mathtt{if}\ B\ \mathtt{then}\ S_t\ \mathtt{else}\ S_f\ \mathtt{fi}\rrbracket] \circ \ddot{\gamma}\llbracket P\rrbracket$$

$=$ $\wr$big step operational semantics (93)$\wr$

$$\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{post}\,[(1_{\Sigma\llbracket P\rrbracket} \cup \tau^{B}) \circ \tau^\star\llbracket S_t\rrbracket \circ (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{t}) \cup (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{\bar{B}}) \circ \tau^\star\llbracket S_f\rrbracket \circ (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{f})] \circ \ddot{\gamma}\llbracket P\rrbracket$$

$=$ $\wr$Galois connection (98) so that post preserves joins$\wr$

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$=$ $\wr$Galois connection (106) so that $\ddot{\alpha}\llbracket P\rrbracket$ preserves joins$\wr$

$$(\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{post}[(1_{\Sigma\llbracket P\rrbracket} \cup \tau^{B}) \circ \tau^\star\llbracket S_t\rrbracket \circ (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{t})] \circ \ddot{\gamma}\llbracket P\rrbracket) \mathbin{\ddot{\sqcup}} (\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{post}[(1_{\Sigma\llbracket P\rrbracket} \cup \tau^{\bar{B}}) \circ \tau^\star\llbracket S_f\rrbracket \circ (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{f})] \circ \ddot{\gamma}\llbracket P\rrbracket)$$

$\ddot{\sqsubseteq}$ $\wr$lemma (5.3) and similar one for the **else** branch$\wr$

```
λJ • let J^{t'} = λl ∈ in_P⟦P⟧•(l = at_P⟦S_t⟧ ? J_{at_P⟦S_t⟧} ⊔̇ Abexp⟦B⟧(J_ℓ) ¿ J_l) in        (120)
       let J^{t''} = APost⟦S_t⟧(J^{t'}) in
         λl ∈ in_P⟦P⟧•(l = ℓ' ? J^{t''}_{ℓ'} ⊔̇ J^{t''}_{after_P⟦S_t⟧} ¿ J^{t''}_l)
     ⊔̈
     let J^{f'} = λl ∈ in_P⟦P⟧•(l = at_P⟦S_f⟧ ? J_{at_P⟦S_f⟧} ⊔̇ Abexp⟦T(¬B)⟧(J_ℓ) ¿ J_l) in
       let J^{f''} = APost⟦S_f⟧(J^{f'}) in
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```

$=$ $\wr$by grouping similar terms$\wr$

```
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     and J^{f'} = λl ∈ in_P⟦P⟧•(l = at_P⟦S_f⟧ ? J_{at_P⟦S_f⟧} ⊔̇ Abexp⟦T(¬B)⟧(J_ℓ) ¿ J_l) in
       let J^{t''} = APost⟦S_t⟧(J^{t'})
       and J^{f''} = APost⟦S_f⟧(J^{f'}) in
         λl ∈ in_P⟦P⟧•(l = ℓ' ? J^{t''}_{ℓ'} ⊔̇ J^{t''}_{after_P⟦S_t⟧} ⊔̇ J^{f''}_{ℓ'} ⊔̇ J^{f''}_{after_P⟦S_f⟧} ¿ J^{t''}_l ⊔̇ J^{f''}_l)
```

$=$ $\wr$by locality (113) and labelling scheme (59) so that in particular $J^{t''}_{\ell'} = J^{t'}_{\ell'} = J^{t}_{\ell'} = J^{f}_{\ell'} = J^{f'}_{\ell'} = J^{f''}_{\ell'}$ and $\mathrm{APost}\llbracket S_t\rrbracket$ and $\mathrm{APost}\llbracket S_f\rrbracket$ do not interfere$\wr$

Formalization part

$$\ddot{\alpha}\llbracket P\rrbracket(\mathrm{Post}\llbracket \mathtt{if}\ B\ \mathtt{then}\ S_t\ \mathtt{else}\ S_f\ \mathtt{fi}\rrbracket)$$

$= \quad$ ⦃def. (110) of $\ddot{\alpha}\llbracket P\rrbracket$⦄

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$$(\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{post}[(1_{\Sigma\llbracket P\rrbracket} \cup \tau^{B}) \circ \tau^{\star}\llbracket S_t\rrbracket \circ (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{t})] \circ \ddot{\gamma}\llbracket P\rrbracket) \mathbin{\ddot{\cup}} (\ddot{\alpha}\llbracket P\rrbracket \circ \mathrm{post}[(1_{\Sigma\llbracket P\rrbracket} \cup \tau^{\bar{B}}) \circ \tau^{\star}\llbracket S_f\rrbracket \circ (1_{\Sigma\llbracket P\rrbracket} \cup \tau^{f})] \circ \ddot{\gamma}\llbracket P\rrbracket)$$

$\ddot{\sqsubseteq} \quad$ ⦃lemma (5.3) and similar one for the **else** branch⦄

$$\begin{aligned}
&\lambda J\cdot \mathrm{let}\ J^{t'} = \lambda l \in \mathrm{in}_P\llbracket P\rrbracket\cdot(l = \mathrm{at}_P\llbracket S_t\rrbracket\ ?\ J_{\mathrm{at}_P\llbracket S_t\rrbracket} \mathbin{\dot{\sqcup}} \mathrm{Abexp}\llbracket B\rrbracket(J_\ell)\ \text{¿}\ J_l)\ \mathrm{in} && (120)\\
&\quad \mathrm{let}\ J^{t''} = \mathrm{APost}\llbracket S_t\rrbracket(J^{t'})\ \mathrm{in}\\
&\quad\quad \lambda l \in \mathrm{in}_P\llbracket P\rrbracket\cdot(l = \ell'\ ?\ J^{t''}_{\ell'} \mathbin{\dot{\sqcup}} J^{t''}_{\mathrm{after}_P\llbracket S_t\rrbracket}\ \text{¿}\ J^{t''}_l)\\
&\quad \ddot{\sqcup}\\
&\quad \mathrm{let}\ J^{f'} = \lambda l \in \mathrm{in}_P\llbracket P\rrbracket\cdot(l = \mathrm{at}_P\llbracket S_f\rrbracket\ ?\ J_{\mathrm{at}_P\llbracket S_f\rrbracket} \mathbin{\dot{\sqcup}} \mathrm{Abexp}\llbracket T(\neg B)\rrbracket(J_\ell)\ \text{¿}\ J_l)\ \mathrm{in}\\
&\quad\quad \mathrm{let}\ J^{f''} = \mathrm{APost}\llbracket S_f\rrbracket(J^{f'})\ \mathrm{in}\\
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\end{aligned}$$

$= \quad$ ⦃by grouping similar terms⦄

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\end{aligned}$$

$= \quad$ ⦃by locality (113) and labelling scheme (59) so that in particular $J^{t''}_{\ell'} = J^{t'}_{\ell'} = J^{t}_{\ell'} = J^{f}_{\ell'} = J^{f'}_{\ell'} = J^{f''}_{\ell'}$ and $\mathrm{APost}\llbracket S_t\rrbracket$ and $\mathrm{APost}\llbracket S_f\rrbracket$ do not interfere⦄

Implementation part

```
int main(int argc, char **argv)
{
  int i, j, t, parent_a, parent_b;
  int **swap, **newpop, **oldpop;
  double *fit, *normfit;

  get_options(argc, argv, options, help_string);
  srandom(seed);
  read_specs(specs);
  size += (size / 2 * 2 != size);
  newpop = xmalloc(sizeof(int *) * size);
  oldpop = xmalloc(sizeof(int *) * size);
  fit = xmalloc(sizeof(double) * size);
  normfit = xmalloc(sizeof(double) * size);
  for(i = 0; i < size; i++) {
    newpop[i] = xmalloc(sizeof(int) * len);
    oldpop[i] = xmalloc(sizeof(int) * len);
    for(j = 0; j < len * 2; j++)
      random_solution(oldpop[i]);
  }
  for(t = 0; t < gens; t++) {
    compute_fitness(oldpop, fit, normfit);
    dump_stats(t, oldpop, fit);
    for(i = 0; i < size; i += 2) {
      parent_a = select_one(normfit);
      parent_b = select_one(normfit);
      reproduce(oldpop, newpop, parent_a, parent_b, i);
    }
    swap = newpop; newpop = oldpop; oldpop = swap;
  }
  exit(0);
}
```

Formalization part

$\dot{\ddot{\alpha}}[\![P]\!](\text{Post}[\![\texttt{if } B \texttt{ then } S_t \texttt{ else } S_f \texttt{ fi}]\!])$

$=$ ⟅def. (110) of $\dot{\ddot{\alpha}}[\![P]\!]$⟆

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$\ddot{\alpha}[\![P]\!] \circ \text{post}\,[(1_{\Sigma[\![P]\!]} \cup \tau^{B}) \circ \tau^\star[\![S_t]\!] \circ (1_{\Sigma[\![P]\!]} \cup \tau^{t}) \cup (1_{\Sigma[\![P]\!]} \cup \tau^{\bar{B}}) \circ \tau^\star[\![S_f]\!] \circ (1_{\Sigma[\![P]\!]} \cup \tau^{f})] \circ \ddot{\gamma}[\![P]\!]$

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$=$

$(\ddot{\alpha}[$

pos

$\dot{\sqsubseteq}$ ⟅I

$\lambda J \cdot \text{let } J^{t'} = \lambda l \in \text{in}_P[\![P]\!] \cdot (l = \text{at}_P[\![S_t]\!] \;?\; J_{\text{at}_P[\![S_t]\!]} \mathbin{\dot{\sqcup}} \text{Abexp}[\![B]\!](J_\ell) \;¿\; J_l) \text{ in}$ (120)

$\quad \text{let } J^{t''} = \text{APost}[\![S_t]\!](J^{t'}) \text{ in}$

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Ü

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Implementation part

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  int i, j, t, parent_a, parent_b;
  int **swap, **newpop, **oldpop;
  double *fit, *normfit;

  get_options(argc, argv, options, help_string);
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  read_specs(specs);
  size += (size / 2 * 2 != size);
  newpop = xmalloc(sizeof(int *) * size);
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    oldpop[i] = xmalloc(sizeof(int) * len);
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    swap = newpop; newpop = oldpop; oldpop = swap;
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```

Do both parts talk about the same ?

Static Analysis for real-life languages

Example of real-life language : bytecode JavaCard

- 180 instructions
- Real need of static analysis to verify properties about security, memory management, ...

For this kind of languages,

- Abstract domains can be complex
- Correctness proofs become long and tiresome
- Implementation and maintenance of the analyser become a software engineering task

In this work

We propose a technique based on the Coq proof assistant

- To specify a static analysis,
- To prove its correctness wrt. the semantics of the language,
- To extract a static analyser from the proof of existence of a correct program analysis result

Program-as-proofs paradigm:

Write a function f which verifies a specification P $\forall x,\ P(x, f(x))$ $\Longleftrightarrow$ Make a constructive proof of $\forall x,\ \exists y,\ P(x, y)$

Outline

- Motivation
- A Static Analysis for Carmel
- Building a certified static analyser
- Conclusion

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Case study : a static analysis for Carmel

We follow the analysis proposed by René Rydhof Hansen[1]

- Carmel : an intermediate representation of Java Card byte code
- Construction of a certified data flow analyser for Carmel

[1] René Rydhof Hansen. Flow Logic for Carmel. SECSAFE-IMM-001, 2002

Syntax of Carmel

Instruction $::=$

$$\begin{array}{l}\texttt{nop}\end{array}$$

$$\left.\begin{array}{l}\texttt{push}\ c\\ \texttt{pop}\\ \texttt{numop}\ op\end{array}\right\}\text{stack manipulation}$$

$$\left.\begin{array}{l}\texttt{load}\ x\\ \texttt{store}\ x\end{array}\right\}\text{local variables manipulation}$$

$$\left.\begin{array}{l}\texttt{if}\ pc\\ \texttt{goto}\ pc\end{array}\right\}\text{jump}$$

$$\left.\begin{array}{l}\texttt{new}\ cl\\ \texttt{putfield}\ f\\ \texttt{getfield}\ f\end{array}\right\}\text{heap manipulation}$$

$$\left.\begin{array}{l}\texttt{invokevirtual}\ m_{id}\\ \texttt{return}\end{array}\right\}\text{method call and return}$$

Semantic domains

$$
\begin{array}{rcll}
\mathsf{Val} & ::= & \mathsf{num}\ n & n \in \mathbb{N} \\
 & & \mathsf{ref}\ r & r \in \mathsf{Reference} \\
 & & \mathsf{null} & \\
\mathsf{Stack} & = & \mathsf{Val}^* & \\
\mathsf{LocalVar} & = & \mathsf{Var} \rightarrow \mathsf{Val} & \\
\mathsf{Frame} & = & \mathsf{PointProg} \times \mathsf{NameMethod} & \\
 & & & \times \mathsf{LocalVar} \times \mathsf{Stack} \\
\mathsf{CallStack} & = & \mathsf{Frame}^* & \\
\mathsf{Object} & = & \mathsf{FieldName} \rightarrow \mathsf{Val} & \\
\mathsf{Heap} & = & \mathsf{Reference} \rightarrow \mathsf{Object}_{\perp} & \\
\mathsf{State} & = & \mathsf{Heap} \times \mathsf{CallStack} &
\end{array}
$$

Example :

$$(H, \langle m, pc, L, v :: S \rangle :: SF)$$

Dynamic semantics

Operational semantics with rules like

$$\frac{\text{instructionAt}_P(m, pc) = \texttt{push}\ c}{(H, \langle m, pc, L, S\rangle :: SF) \Rightarrow (H, \langle m, pc+1, L, c :: S\rangle :: SF)}$$

$$\frac{\begin{array}{c}\text{instructionAt}_P(m, pc) = \texttt{invokevirtual}\ m_{id} \\ \begin{array}{rcl} m' & = & \text{methodLookup}(m_{id}, h(loc)) \\ f' & = & \langle m', 1, V, \varepsilon\rangle \\ f'' & = & \langle m, pc, l, s\rangle \end{array}\end{array}}{(h, \langle m, pc, l, loc :: V :: s\rangle :: sf) \Rightarrow (h, f' :: f'' :: sf)}$$

A Static Analysis for Carmel

We want to calculate an approximation $\left(\hat{H}, \hat{L}, \hat{S}\right)$ on the domain

$$\begin{aligned}\widehat{\text{State}} = \widehat{\text{Heap}} \quad & \times \quad \left(\text{NameMethod} \times \text{PointProg} \rightarrow \widehat{\text{LocalVar}}\right) \\ & \times \quad \left(\text{NameMethod} \times \text{PointProg} \rightarrow \widehat{\text{Stack}}\right)\end{aligned}$$

- An approximation for all reachable heaps
- For each program points, an approximation of the operand stack and the local variables
- An object is abstracted to its class
- Numeric values are abstracted using Killdall’s Constant Propagation domain

Analysis specification

Each instruction impose constraints on $\left(\hat{H}, \hat{L}, \hat{S}\right)$.

Example

```
0 :   push 1
1 :   push 2
2 :   store 0
3 :   load 0
4 :   numop mult
5 :   goto 1
```

Analysis specification

Each instruction impose constraints on $\left(\hat{H}, \hat{L}, \hat{S}\right)$.

Example

$\widehat{\mathrm{nil}} \sqsubseteq \hat{S}(m, 0)$ $\top \sqsubseteq \hat{L}(m, 0)$

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```

$$\widehat{\text{nil}} \sqsubseteq \hat{S}(m, 0) \qquad \top \sqsubseteq \hat{L}(m, 0)$$

$$\widehat{\text{push}}(\hat{1}, \hat{S}(m, 0)) \sqsubseteq \hat{S}(m, 1) \qquad \hat{L}(m, 0) \sqsubseteq \hat{L}(m, 1)$$

Analysis specification

Each instruction impose constraints on $\left(\hat{H}, \hat{L}, \hat{S}\right)$.

Example

	$\widehat{\text{nil}} \sqsubseteq \hat{S}(m,0)$	$\top \sqsubseteq \hat{L}(m,0)$
0 : `push 1`	$\widehat{\text{push}}(\hat{1}, \hat{S}(m,0)) \sqsubseteq \hat{S}(m,1)$	$\hat{L}(m,0) \sqsubseteq \hat{L}(m,1)$
1 : `push 2`	$\widehat{\text{push}}(\hat{2}, \hat{S}(m,1)) \sqsubseteq \hat{S}(m,2)$	$\hat{L}(m,1) \sqsubseteq \hat{L}(m,2)$
2 : `store 0`	$\widehat{\text{pop}}(\hat{S}(m,2)) \sqsubseteq \hat{S}(m,3)$	$\hat{L}(m,2)[0 \mapsto \widehat{\text{top}}(\hat{S}(m,2))] \sqsubseteq \hat{L}(m,3)$
3 : `load 0`	$\widehat{\text{push}}(\hat{L}(m,3)[0], \hat{S}(m,3)) \sqsubseteq \hat{S}(m,4)$	$\hat{L}(m,3) \sqsubseteq \hat{L}(m,4)$
4 : `numop mult`	$\ldots$	$\hat{L}(m,4) \sqsubseteq \hat{L}(m,5)$
5 : `goto 1`	$\hat{S}(m,5) \sqsubseteq \hat{S}(m,1)$	$\hat{L}(m,5) \sqsubseteq \hat{L}(m,1)$

Analysis solution

The smallest value which verifies all constraints.

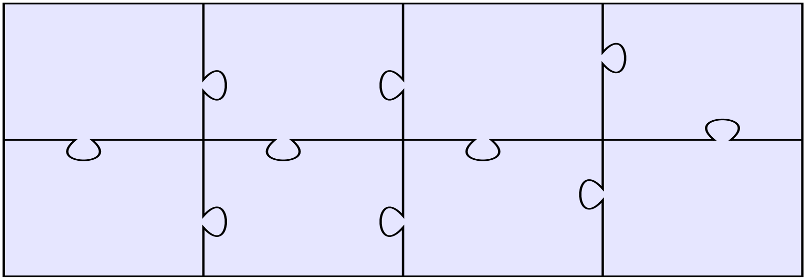
Example

		$\widehat{\text{nil}}$	$[0 \mapsto \top;\ 1 \mapsto \top]$
0 :	`push 1`		
		$< \hat{2} >$	$[0 \mapsto \hat{1};\ 1 \mapsto \top]$
1 :	`push 2`		
		$< \hat{1} :: \hat{2} >$	$[0 \mapsto \hat{1};\ 1 \mapsto \top]$
2 :	`store 0`		
		$< \hat{2} >$	$[0 \mapsto \hat{1};\ 1 \mapsto \top]$
3 :	`load 0`		
		$< \hat{1} :: \hat{2} >$	$[0 \mapsto \hat{1};\ 1 \mapsto \top]$
4 :	`numop mult`		
		$< \hat{2} >$	$[0 \mapsto \hat{1};\ 1 \mapsto \top]$
5 :	`goto 1`		
		$\perp$	$\perp$

Outline

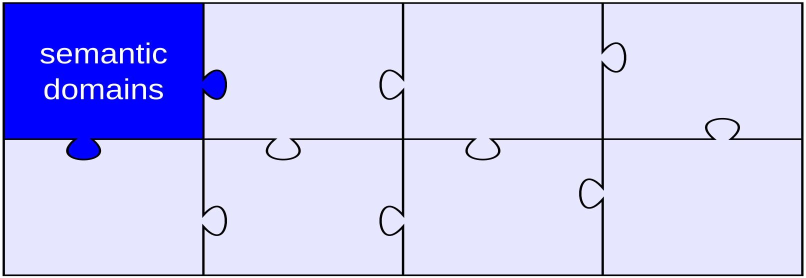
- Motivation
- A Static Analysis for Carmel
- Building a certified static analyser
- Conclusion

Building a certified static analyser



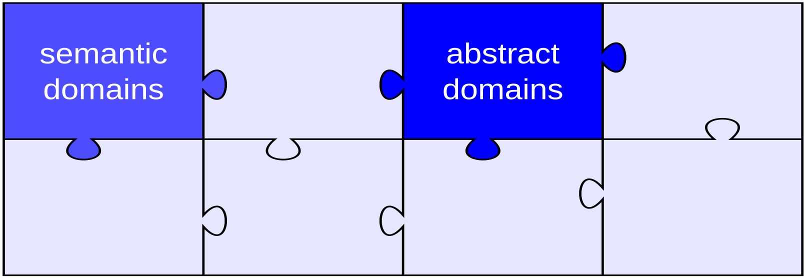
- A puzzle with 8 pieces,
- Each piece interacts with its neighbors

Building a certified static analyser



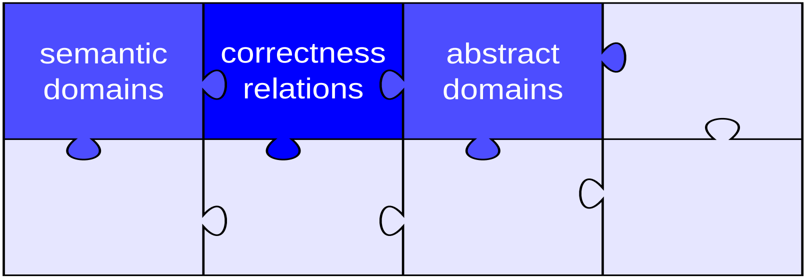
- Each semantic domain is modeled with a type
- Following exactly the definitions already seen in a previous slide

Building a certified static analyser



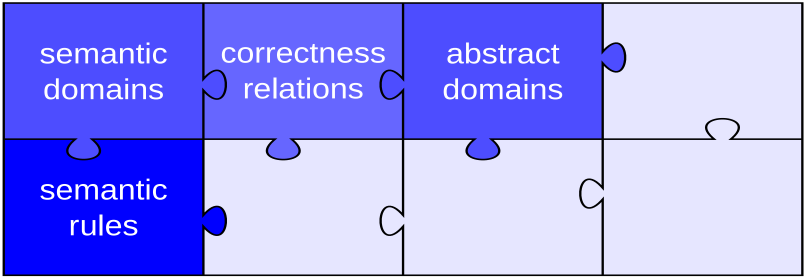
- Each semantic domain is in relation with an abstract domain
- an abstract domain is a lattice (formalization of lattices in Coq to follow...)

Building a certified static analyser



- A relation $\sim$ between State and $\widehat{\text{State}}$
- $s \sim \widehat{\Sigma}$ interprets as "$\widehat{\Sigma}$ is a correct approximation of s"
- $\sim$ must be monotone :
  $\forall s \in \text{State}, \forall \widehat{\Sigma}_1, \widehat{\Sigma}_2 \in \widehat{\text{State}},$
  $\quad$ if $s \sim \widehat{\Sigma}_1$ and $\widehat{\Sigma}_1 \sqsubseteq \widehat{\Sigma}_2$ then $s \sim \widehat{\Sigma}_2$

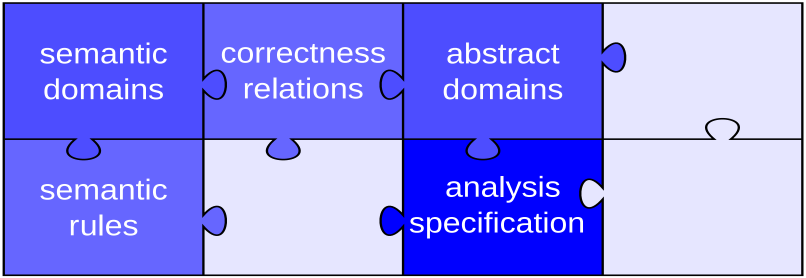
Building a certified static analyser



- The transition relation $\cdot \Rightarrow \cdot$ is defined using Coq inductive types
- Collecting semantics :
  $[\![P]\!] = \{s \mid \exists s_0 \text{ an initial state}, \text{with } s_0 \Rightarrow^* s\}$

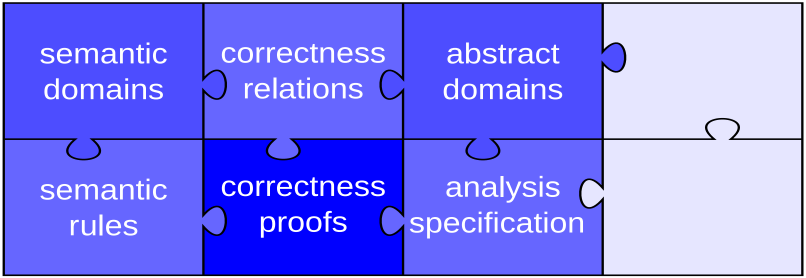
We want to compute a correct approximation of $[\![P]\!]$

Building a certified static analyser



- we define a predicate $P \vdash \widehat{\Sigma}$ which imposes a set of constraints on an abstract state $\widehat{\Sigma}$

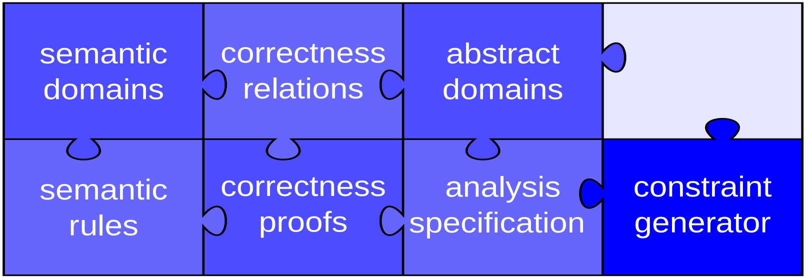
Building a certified static analyser



$$\forall P : \text{Program},\ \forall \widehat{\Sigma} : \widehat{\text{State}},\quad P \vdash \widehat{\Sigma} \Rightarrow [\![P]\!] \sim \widehat{\Sigma}$$

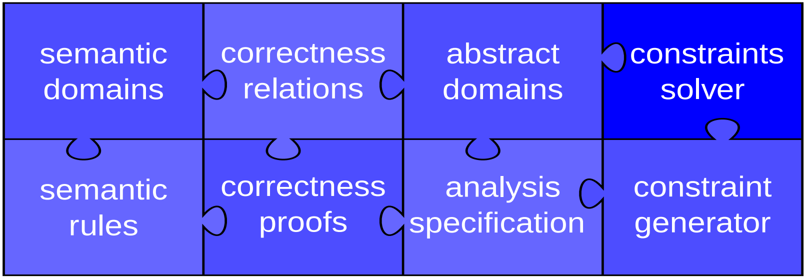
- One case by instruction
- With a special treatment for the `invokevirtual`/`return` instructions

Building a certified static analyser



- Collects all constraint of a given program

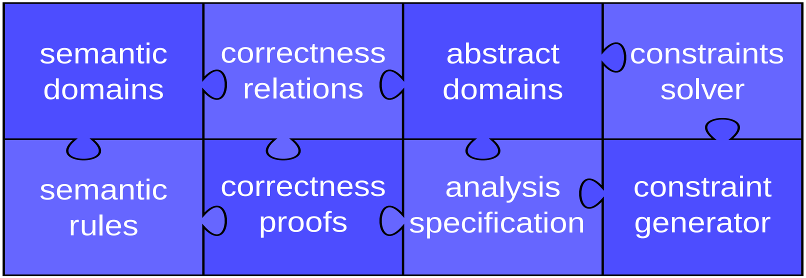
Building a certified static analyser



$$\forall P : \text{Program},\ \exists \widehat{\Sigma} : \widehat{\text{State}},\quad P \vdash \widehat{\Sigma}$$

- In fact, a stronger result : there exists a smallest solution

Building a certified static analyser



Final result

$$\left.\begin{array}{l}\forall P,\ \forall\widehat{\Sigma},\ P\vdash\widehat{\Sigma}\Rightarrow[\![P]\!]\sim\widehat{\Sigma}\\ \forall P,\ \exists\widehat{\Sigma},\ P\vdash\widehat{\Sigma}\end{array}\right\}\quad \forall P,\ \exists\widehat{\Sigma},\ [\![P]\!]\sim\widehat{\Sigma}$$

Abstract domains

The `lattice` type is a big structure :

```
Record Lattice [A:Set] :  Type := {
  eq :  A → A → Prop;

  order :  A → A → Prop;

  join :  A → A → A;

  eq_dec :  A → A → bool;

  bottom :  A;

  top :  A;


}.
```

Abstract domains

The `lattice` type is a big structure :

```
Record Lattice [A:Set] :  Type := {
  eq :  A → A → Prop;
    eq_prop :  ...; // eq  is an equivalence relation
  order :  A → A → Prop;
    order_prop :  ...; // order  is an order relation
  join :  A → A → A;
    join_prop :  ...; // join  is a correct binary least upper bound
  eq_dec :  A → A → bool;
    eq_dec_prop :  ...; // eq_dec  is a correct equality test
  bottom :  A;
    bottom_prop :  ...; // bottom  is the smallest element
  top :  A;
    top_prop :  ...; // top  is the biggest element
    acc_prop :  ...; // ⊐  is well founded (ascending chain condition)
}.
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Abstract domains

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    top_prop : ...; // top  is the biggest element
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}.
```

A lattice library

- Two base lattices
  - Flat lattice of constants
  - Lattice of sets over a finite subset of integer
- Four functions to combine lattices
  - Product of lattice
  - Sum of lattice
  - Arrays whose elements live in a lattice and whose size is bounded (efficient functional structure)
  - List whose elements live in a lattice

This modular construction saves a considerable amount of time and effort.

For this analysis : (array (array (list (finiteSet $+$ constants))))
$\times$ (array (array (array (finiteSet $+$ constants))))
$\times$ (array (array (finiteSet $+$ constants)))

Constraint solver

1. A generic fixed point solver

$$\forall L : (\texttt{lattice}\ A),\ \forall f : A \to A,\ f \text{ monotone},$$
$$\exists x : A,\ x \text{ is the least fixed point of } f$$

   proof : the sequence $\bot, f(\bot), f^2(\bot), \ldots$ stabilizes on the least fixed point

2. We use it to solve the functional constraints, using the fact that

$$\begin{array}{c} x \text{ is the least solution of} \\ f_1(x) \sqsubseteq x, \ldots, f_n(x) \sqsubseteq x \end{array} \iff \begin{array}{c} x \text{ is the least fixed point of} \\ \hat{f}_1 \circ \cdots \circ \hat{f}_n \end{array}$$

   with $\hat{f}(x) = f(x) \sqcup x$

3. Combined with the constraint generator, we obtain

$$\forall P,\ \exists \widehat{\Sigma},\ P \vdash \widehat{\Sigma}$$

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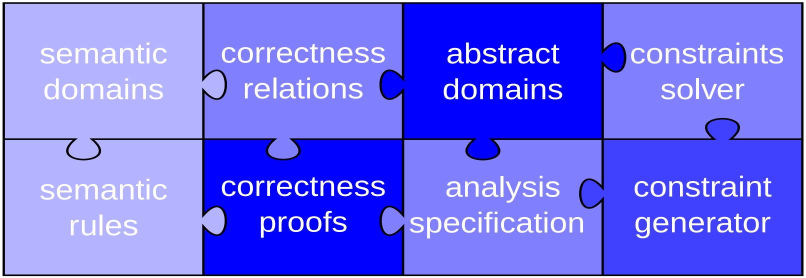
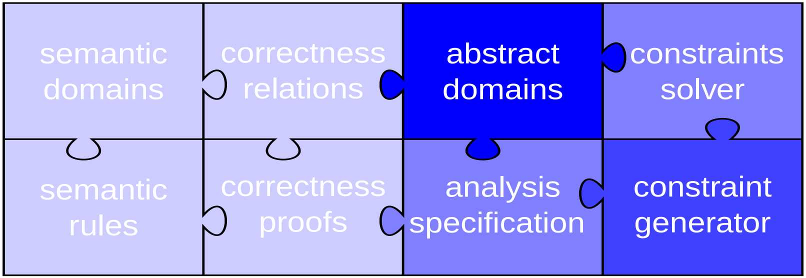
Where is the proof effort ?



- Most technical part in the lattice library
- Correctness part does not require specific competences in Coq
- A majority of proof a reusable to develop others analysis

Where is the programming effort ?



- The extraction mechanism only keeps the computational content of proofs
- The corresponding parts require a high attention to obtain an efficient analyser

Conclusion on the work

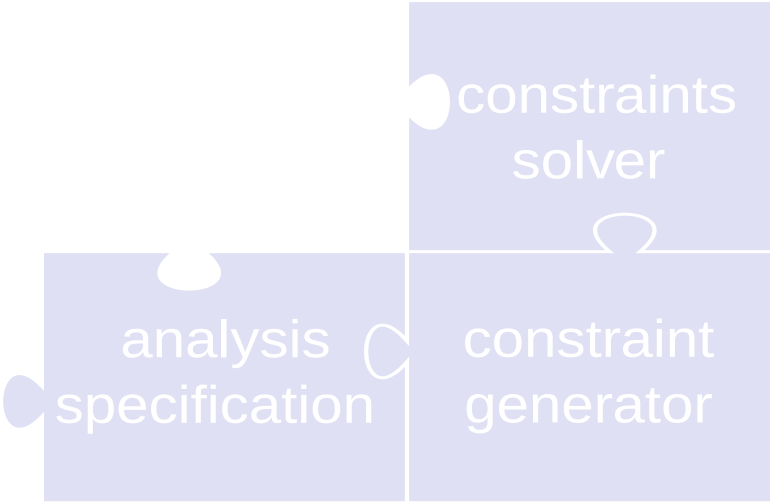
- We proposed a technique based on the Coq proof assistant
  - To develop a certified static analyser
  - To extract a correct analyser in Ocaml
- We illustrated this technique with a data flow analysis for the Carmel language
  - 10000 lines of Coq converted in 2000 lines of OCaml
  - With a reasonable efficiency of the analyser :
    - About 1 minute to analyse 1000 lines of Carmel byte code

Further works

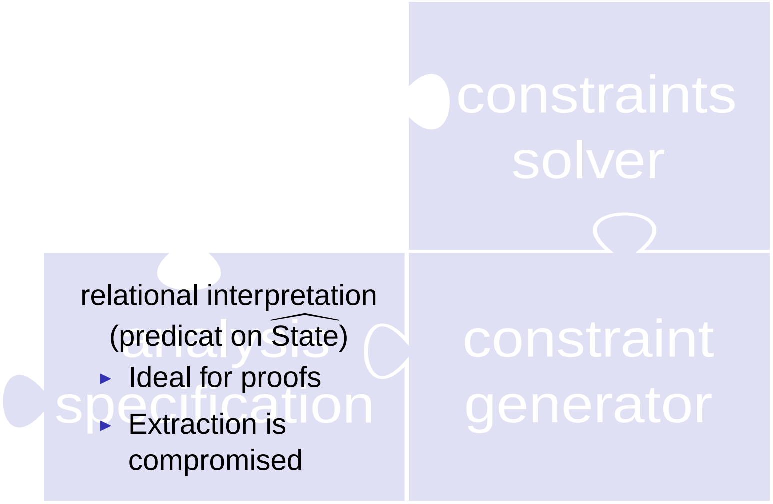
- Construction of an efficient certified work-set based program instead of the actual naive resolution
  - This program must be independent of the abstract domains
- Lattice of infinite height like intervals
- Automatization of the correctness proof ?
  - So as to quickly extend the number of language instructions
- A more extensive use of the abstract interpretation formalism
  - We must find a compromise between the reusability possibilities and the technical efforts in Coq
- Application of this technique to others languages

?

Constraint representation



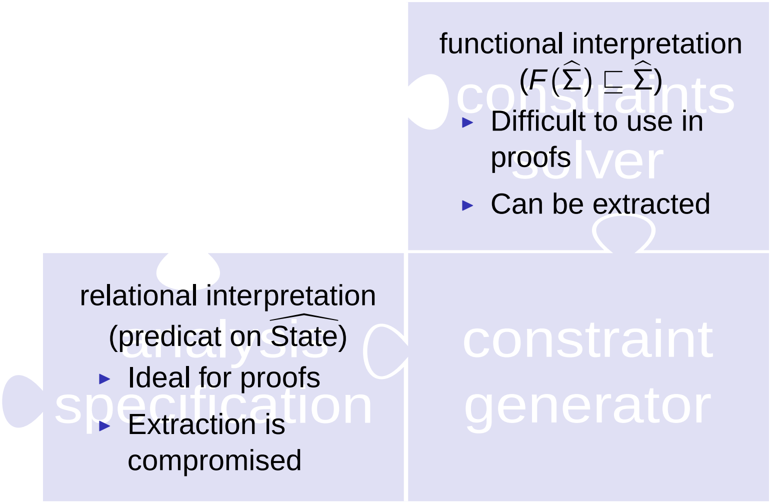
Constraint representation



relational interpretation
(predicat on $\widehat{\text{State}}$)

- Ideal for proofs
- Extraction is compromised

Constraint representation



functional interpretation
$(F(\widehat{\Sigma}) \sqsubseteq \widehat{\Sigma})$

- Difficult to use in proofs
- Can be extracted

relational interpretation
(predicat on $\widehat{\text{State}}$)

- Ideal for proofs
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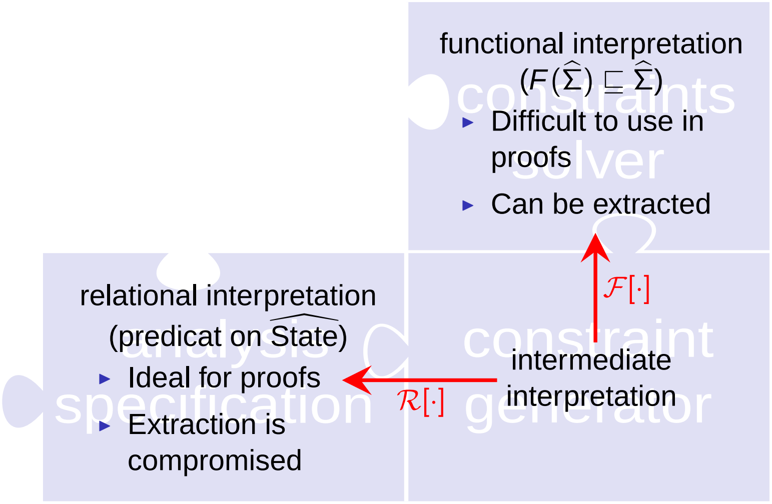
Constraint representation



functional interpretation
$(F(\widehat{\Sigma}) \sqsubseteq \widehat{\Sigma})$

- Difficult to use in proofs
- Can be extracted

intermediate interpretation

relational interpretation
(predicat on $\widehat{\text{State}}$)

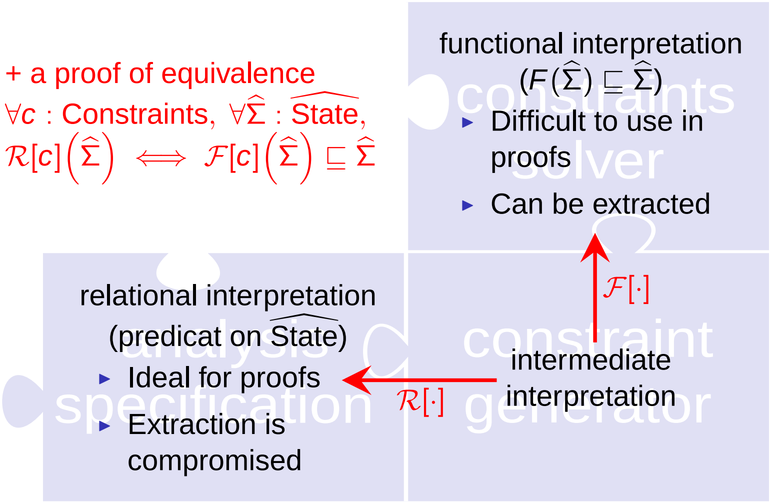
- Ideal for proofs
- Extraction is compromised

Constraint representation

+ a proof of equivalence

$\forall c : \text{Constraints}, \ \forall \widehat{\Sigma} : \widehat{\text{State}},$

$\mathcal{R}[c]\left(\widehat{\Sigma}\right) \iff \mathcal{F}[c]\left(\widehat{\Sigma}\right) \sqsubseteq \widehat{\Sigma}$



functional interpretation
($F(\widehat{\Sigma}) \sqsubseteq \widehat{\Sigma}$)

- Difficult to use in proofs
- Can be extracted

relational interpretation
(predicat on $\widehat{\text{State}}$)

- Ideal for proofs
- Extraction is compromised

intermediate interpretation

Correctness proof

$$\forall P : \text{Program},\ \forall \widehat{\Sigma} : \widehat{\text{State}},\quad P \vdash \widehat{\Sigma} \Longrightarrow [\![P]\!] \sim \widehat{\Sigma}$$

We prove it by well-founded induction on the length of the program execution.
Induction step:

- for all instructions I, except `return`
  $$\begin{array}{l} \forall P,\ \forall \widehat{\Sigma} : \widehat{\text{State}},\ P \vdash \widehat{\Sigma} \Longrightarrow \\ \qquad \forall s_1 : \text{State},\ s_1 \sim \widehat{\Sigma} \Longrightarrow \\ \qquad\qquad \forall s_2 : \text{State},\ [s_1 \Rightarrow_I s_2] \Longrightarrow s_2 \sim \widehat{\Sigma} \end{array}$$
- for the return instruction
  $$\begin{array}{l} \forall P,\ \forall \widehat{\Sigma} : \widehat{\text{State}},\ P \vdash \widehat{\Sigma} \Longrightarrow \\ \qquad \forall s_1 : \text{State},\ \left(\forall s,\ s \Rightarrow^* s_1 \Longrightarrow s \sim \widehat{\Sigma}\right) \Longrightarrow \\ \qquad\qquad \forall s_2 : \text{State},\ [s_1 \Rightarrow_{\texttt{return}} s_2] \Longrightarrow s_2 \sim \widehat{\Sigma} \end{array}$$